WORKSHOP CALCULATION & SCIENCE

(NSQF)

1st YEAR

(As per Revised Syllabus July 2022)

Information Technology Support Executive

Directorate General of Training

DIRECTORATE GENERAL OF TRAINING
MINISTRY OF SKILL DEVELOPMENT & ENTREPRENEURSHIP
GOVERNMENTOF INDIA

NATIONAL INSTRUCTIONAL MEDIA INSTITUTE, CHENNAI

Post Box No. 3142, CTI Campus, Guindy, Chennai - 600 032

Workshop Calculation & Science

Information Technology Support Executive - 1st Year NSQF

As per Revised Syllabus July 2022

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FOREWORD

The Government of India has set an ambitious target of imparting skills to 30 crores people, one out of every four Indians, by 2020 to help them secure jobs as part of the National Skills Development Policy. Industrial Training Institutes (ITIs) play a vital role in this process especially in terms of providing skilled manpower. Keeping this in mind, and for providing the current industry relevant skill training to Trainees, ITI syllabus has been recently updated with the help of comprising various stakeholder's viz. Industries, Entrepreneurs, Academicians and representatives from ITIs.

The National Instructional Media Institute (NIMI), Chennai, has now come up with instructional material to suit the revised curriculum for **Workshop Calculation & Science - Information Technology Support Executive 1st Year** NSQF (Revised 2022) under CTS will help the trainees to get an international equivalency standard where their skill proficiency and competency will be duly recognized across the globe and this will also increase the scope of recognition of prior learning. NSQF trainees will also get the opportunities to promote life long learning and skill development. I have no doubt that with NSQF the trainers and trainees of ITIs, and all stakeholders will derive maximum benefits from these IMPs and that NIMI's effort will go a long way in improving the quality of Vocational training in the country.

The Executive Director & Staff of NIMI and members of Media Development Committee deserve appreciation for their contribution in bringing out this publication.

Jai Hind

Ms. TRISHALJIT SETHI,
Additional Secretary / Director General (Training),
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New Delhi - 110 001

PREFACE

The National Instructional Media Institute(NIMI) was set up at Chennai, by the Directorate General of Training, Ministry of skill Development and Entrepreneurship, Government of India, with the technical assistance from the Govt of the Federal Republic of Germany with the prime objective of developing and disseminating instructional Material for various trades as per prescribed syllabus and Craftsman Training Programme(CTS) under NSQF levels.

The Instructional materials are developed and produced in the form of Instructional Media Packages (IMPs), consisting of Trade Theory, Trade Practical, Test and Assignment Book, Instructor Guide, Wall charts, Transparencies and other supportive materials. The above material will enable to achieve overall improvement in the standard of training in ITIs.

A national multi-skill programme called SKILL INDIA, was launched by the Government of India, through a Gazette Notification from the Ministry of Finance (Dept of Economic Affairs), Govt of India, dated 27th December 2013, with a view to create opportunities, space and scope for the development of talents of Indian Youth, and to develop those sectors under Skill Development.

The emphasis is to skill the Youth in such a manner to enable them to get employment and also improve Entrepreneurship by providing training, support and guidance for all occupation that were of traditional types. The training programme would be in the lines of International level, so that youths of our Country can get employed within the Country or Overseas employment. The **National Skill Qualification Framework (NSQF)**, anchored at the National Skill Development Agency(NSDA), is a Nationally Integrated Education and competency-based framework, to organize all qualifications according to a series of **levels of Knowledge, Skill and Aptitude.** Under NSQF the learner can acquire the Certification for Competency needed at any level through formal, non-formal or informal learning.

The **Workshop Calculation & Science -** Information Technology Support Executive 1st Year NSQF (Revised 2022) under CTS is one of the book developed by the core group members as per the NSQF syllabus.

The **Workshop Calculation & Science -** Information Technology Support Executive 1st Year NSQF (Revised 2022) under CTS as per NSQF is the outcome of the collective efforts of experts from Field Institutes of DGT, Champion ITI's for each of the Sectors, and also Media Development Committee **(MDC)** members and Staff of **NIMI.** NIMI wishes that the above material will fulfill to satisfy the long needs of the trainees and instructors and shall help the trainees for their Employability in Vocational Training.

NIMI would like to take this opportunity to convey sincere thanks to all the Members and Media Development Committee (MDC) members.

Chennai - 600 032 **EXECUTIVE DIRECTOR**

ACKNOWLEDGEMENT

The National Instructional Media Institute (NIMI) sincerely acknowledge with thanks the co-operation and contribution of the following Media Developers to bring this IMP for the course **Workshop Calculation & Science - Information Technology Support Executive 1st Year** as per NSQF Revised 2022.

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NIMI CO-ORDINATORS

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Shri. G. Michael Johny	-	Manager, NIMI, Chennai - 32.

NIMI records its appreciation of the **Data Entry, CAD, DTP Operators** for their excellent and devoted services in the process of development of this IMP.

NIMI also acknowledges with thanks, the efforts rendered by all other staff who have contributed for the development of this book.

INTRODUCTION

The material has been divided into independent learning units, each consisting of a summary of the topic and an assignment part. The summary explains in a clear and easily understandable fashion the essence of the mathematical and scientific principles. This must not be treated as a replacment for the instructor's explanatory information to be imparted to the trainees in the classroom, which certainly will be more elaborate. The book should enable the trainees in grasping the essentials from the elaboration made by the instructor and will help them to solve independently the assignments of the respective chapters. It will also help them to solve the various problems, they may come across on the shop floor while doing their practical exercises.

The assignments are presented through 'Graphics' to ensure communications amongst the trainees. It also assists the trainees to determine the right approach to solve the problems. The required relevent data to solve the problems are provided adjacent to the graphics either by means of symbols or by means of words. The description of the symbols indicated in the problems has its reference in the relevant summaries.

At the end of the exercise wherever necessary assignments, problems are included for further practice.

Time allotment:

Duration of 1st Year : 24 Hrs

Time allotment for each title of exercises has been given below. **Workshop Calculation & Science - Information Technology Support Executive** 1st Year NSQF Revised Syllabus 2022.

S.No	Title	Exercise No.	Time in Hrs
1	Unit, Fractions	1.1.01 - 1.1.07	4
2	Square root, Ratio and Proportions, Percentage	1.2.08 - 1.2.14	6
3	Basic Electricity	1.3.15 - 1.3.18	10
4	Trigonometry	1.4.19 - 1.4.21	4
		Total	**24 Hrs**

LEARNING / ASSESSABLE OUTCOME

On completion of this book you shall be able to

- **Demonstrate basic mathematical concept and principles to perform practical operations.**
- **Understand and explain basic science in the field of study.**

CONTENTS

SYLLABUS

1st Year

Workshop Calculation & Science - Information Technology Support Executive

Revised syllabus July 2022 under CTS

S.No.	Title	Time in Hrs
I	**Unit, Fractions** 1 Classification of Unit System 2 Fundamental and Derived Units F.P.S, C.G.S, M.K.S and SI Units 3 Measurement Units and Conversion 4 Factors, HCF, LCM and Problems 5 Fractions – Addition, Subtraction, Multiplication & Division 6 Decimal Fractions – Addition, Subtraction, Multiplication & Division 7 Solving Problems by using calculator	4
II	**Square root, Ratio and Proportions, Percentage** 1 Square and Square root 2 Simple problems using calculator 3 Applications of Pythagoras theorem and related problems 4 Ratio and Proportion 5 Ratio and Proportion - Direct and Indirect proportions 6 Percentage 7 Percentage - Changing percentage to decimal and fraction	6
III	**Basic Electricity** 1 Introduction and uses of electricity, molecule, atom, how electricity is produced, electric current AC, DC their comparison, voltage, resistance and their units 2 Conductor, Insulator, types of connections - Series and Parallel 3 Ohm's Law, relation between VIR & related problems 4 Electrical power, energy and their units, calculation with assignments	10
iV	**Trigonometry** 1 Measurement of angles 2 Trigonometrical ratios 3 Trigonometrical tables	4
	Total	24

Unit, Fractions - Classification of unit system

Necessity

All physical quantities are to be measured in terms of standard quantities.

Unit

A unit is defined as a standard or fixed quantity of one kind used to measure other quantities of the same kind.

Classification

Fundamental units and derived units are the two classifications.

Fundamental units

Units of basic quantities of length, mass and time.

Derived units

Units which are derived from basic units and bear a constant relationship with the fundamental units.E.g. area, volume, pressure, force etc.

Systems of units

- F.P.S system is the British system in which the basic units of length, mass and time are foot, pound and second respectively.
- C.G.S system is the metric system in which the basic units of length, mass and time are centimeter, gram and seconds respectively.
- M.K.S system is another metric system in which the basic units of length, mass and time are metre, kilogram and second respectively.
- S.I. units are referred to as Systems International units which is again of metric and the basic units, their names and symbols are as follows.

Fundamental units and derived units are the two classifications of units.

Length, mass and time are the fundamental units in all the systems (i.e) F.P.S, C.G.S, M.K.S and S.I. systems.

Example

Length: What is the length of copper wire in the roll , if the roll of copper wire weighs 8kg, the dia of wire is 0.9cm and the density is 8.9 gm/cm^3?

Solution

mass of copper wire in the roll = 8kg (or)8000grams
Dia of copper wire in the roll = 0.9cm
Density of copper wire = 8.9 gm/cm^3

Area of cross section of copper wire

$$= \frac{\pi d^2}{4} = \frac{\pi \times (0.9^2)}{4} = 0.636 cm^2$$

Volume of copper wire

$$= \frac{\text{Mass of copper wire}}{\text{Density of copper wire}} = \frac{8000 grams}{8.9\ gm/cm^3} = 898.88 cm^3$$

Length of copper wire

$$= \frac{\text{Volume of copper wire}}{\text{Area of cross section of copper wire}} = \frac{898.88 cm^3}{0.636 cm^2}$$

$$= 1413.33 \text{ cm}$$

Length of copper wire =1413cm.

Time: The S.I. unit of time, the second, is another base units of S.I., it is defined as the time interval occupied by a number of cycles of radiation from the calcium atom. The second is the same quantity in the S.I. in the British and in the U.S. systems of units.

Fundamental units of F.P.S, C.G.S, M.K.S and S.I

S.No.	Basic quantity	British units		Metric units				International units	
		F.P.S	Symbol	C.G.S	Symbol	M.K.S	Symbol	S.I Units	Symbol
1	Length	Foot	ft	Centimetre	cm	Metre	m	Metre	m
2	Mass	Pound	lb	Gram	g	Kilogram	kg	Kilogram	Kg
3	Time	Second	s	Second	s	Second	s	Second	s
4	Current	Ampere	A	Ampere	A	Ampere	A	Ampere	A
5	Temperature	Fahrenheit	°F	Centigrade	°C	Centigrade	°C	Kelvin	K
6	Light intensity	Candela	Cd	Candela	Cd	Candela	Cd	Candela	Cd

Unit, Fractions - Fundamental and Derived units F.P.S, C.G.S, M.K.S and SI units

Derived units of F.P.S, C.G.S, M.K.S and SI system

S.No	Physical quantity	British units		Metric units				International units	
		FPS	Symbol	CGS	Symbol	MKS	Symbol	SI Units	Symbol
1	Area	Square foot	ft^2	Square centimetre	cm^2	Square metre	m^2	Square metre	m^2
2	Volume	Cubic foot	ft^3	Cubic centimetre	cm^3	Cubic metre	m^3	Cubic metre	m^3
3	Density	Pound per cubic foot	lb/ft^3	Gram per cubic centimetre	g/cm^3	Kilogram per cubic metre	kg/m^3	Kilogram per cubic metre	Kg/m^3
4	Speed	Foot per second	ft/s	Centimetre per second	cm/sec	Metre per second	m/sec	Metre per second	m/sec
5	Velocity (linear)	Foot per second	ft/s	Centimetre per second	cm/sec	Metre per second	m/sec	Metre per second	m/sec
6	Acceleration	Foot per square second	ft/s^2	Centimetre per square second	cm/sec^2	Metre per square second	m/sec^2	Metre per square second	m/sec^2
7	Retardation	Foot per square Second	ft/s^2	Centimetre per square second	cm/sec^2	Metre per square second	m/sec^2	Metre square second	m/sec^2
8	Angular velocity	Degree per second	Deg/sec	Radian per second	rad/sec	Radian per second	rad/sec	Radian per second	rad/sec
9	Mass	Pound (slug)	lb	Gram	g	Kilogram	kg	Kilogram	kg
10	Weight	Pound	lb	Gram	g	Kilogram weight	kg	Newton	N
11	Force	Pounds	lbf	dyne	dyn	Kilogram force	kgf	Newton	$N(kgm/sec^2)$
12	Power	Foot pound per second	ft.lb/sec	Gram.centimetre/sec	g.cm/ sec	kilogram metre per second	kg.m/ sec	-	-
		Horse power	hp	Erg per second		watt	W	watt	W(J/sec)
13	Pressure,Stress	Pound per square inch	lb/in^2	Gram per square centimetre	g/cm^2	Kilogram per square metre	kg/m^2	Newton per square metre	N/m^2
14	Energy, Work	Foot.pound	ft.lb	Gram centimetre	g.cm	Kilogram metre	kg.m	joule	J(Nm)
15	Heat	British thermal unit	BTU	calorie	Cal	joule	J	joule	J(Nm)
16	Torque	Pound force foot	lbf.ft	Newton millimetre	N mm	Kilogram metre	kg.m	Newton metre	Nm
17	Temperature	Degree Fahrenheit	°F	Degree Centigrade	°C	Kelvin	K	Kelvin	K

Unit, Fractions - Measurement units and conversion

Units and abbreviations

Quantity	Units	Abbreviation of unit
Calorific value	kilojoules per kilogram	kJ/kg
Specific fuel consumption	kilogram per hour per newton	kg/hr/N
Length	millimetre, metre, kilometre	mm, m, km
Mass	kilogram, gram	kg, g
Time	seconds, minutes, hours	s, min, h
Speed	centimetre per second, metre per second kilometre per hour, miles per hour	cm/s, m/s km/h, mph
Acceleration	metre-per-square second	m/s^2
Force	newtons, kilonewtons	N,kN
Moment	newton-metres	Nm
Work	joules	J
Power	horsepower, watts, kilowatts	Hp, W, kW
Pressure	newton per square metre kilonewton per square metre	N/m^2 kN/m^2
Angle	radian	rad
Angular speed	radians per second radians-per-square second revolutions per minute revolutions per second	rad/s rad/s^2 Rpm rev/s

Decimal multiples and parts of unit

Decimal power	Value	Prefixes	Symbol	Stands for
10^{12}	1000000000000	tera	T	billion times
10^{9}	1000000000	giga	G	thousand millintimes
10^{6}	1000000	mega	M	million times
10^{3}	1000	kilo	K	thousand times
10^{2}	100	hecto	h	hundred times
10^{1}	10	deca	da	ten times
10^{-1}	0.1	deci	d	tenth
10^{-2}	0.01	centi	c	hundredth
10^{-3}	0.001	milli	m	thousandth
10^{-6}	0.000001	micro	μ	millionth
10^{-9}	0.000000001	nano	n	thousand millionth
10^{-12}	0.000000000001	pico	p	billionth

SI units and the British units:

Quantity	SI unit → British unit	British unit → SI unit
Length	1 m = 3.281 ft 1 km = 0.621 mile	1 ft = 0.3048 m 1 mile = 1.609 km
Speed	1 m/s = 3.281 ft/s 1 km/h = 0.621 mph	1 ft/s = 0.305 m/s 1 mph = 1.61 km/h
Acceleration	1 m/s^2 = 3.281 ft/s^2	1 ft/s^2 = 0.305 m/s^2
Mass	1 kg = 2.205 lb	1 lb = 0.454 kg
Force	1 N = 0.225 lbf 1 MN	1 lbf = 4.448 N 1 million newtons
Torque	1 Nm = 0.738 lbf ft	1 lbf ft = 1.355 Nm
Pressure	1 N/m^2 = 0.000145 lbf/in^2 1 Pa = 1 N/m^2 1 bar = 14.5038 lbf/in^2	1 lbf/in^2 = 6.896 kN/m^2 1 lbf/in^2 = 6.895 kN/m^2
Energy, work	1 J = 0.738 ft lbf 1 J = 0.239 calorie 1 kJ = 0.948 BTU (1 therm = 100 000 BTU) 1 kJ = 0.526 CHU	1 ft lbf = 1.355 J 1 calorie = 4.186 J 1 BTU = 1.055 kJ 1 CHU = 1.9 kJ
Power	1 kW = 1.34 hp	1 hp = 0.7457 kW
Fuel consumption	1km/L = 2.82 mile/gallon	1 mpg = 0.354 km/L
Specific fuel consumption	1 kg/kWh = 1.65 lb/bhp h 1 litre/kWh=1.575 pt/bhp h	1 lb/bhp h = 0.606 kg/kWh 1 pt/bhp h = 0.631 litre/kWh
Calorific value	1 kJ/kg = 0.43 BTU/lb 1 kJ/kg = 0.239 CHU/lb	1 BTU/lb = 2.326 kJ/kg 1 CHU/lb = 4.188 kJ/kg

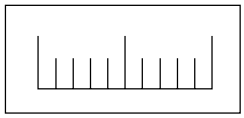
Prefixes for decimal multiples and submultiples

Use		
1 Megapascal	= 1 MPa	= 1000000 Pa
1 Kilowatt	= 1 kW	= 1000 W
1 Hectolitre	= 1 hL	= 100 L
Decanewton	= 1 daN	= 10 N
Decimetre	= 1 dm	= 0.1 m
1 Centimetre	= 1 cm	= 0.01 m
1 Millimetre	= 1 mm	= 0.001 m
1 Micrometre	= 1 um	= 0.000001 m

Conversion factors

1 inch	=	25.4 mm
1 mm	=	0.03937 inch
1 metre	=	39.37 inch
1 micron	=	0.00003937"
1 kilometre	=	0.621 miles
1 pound	=	453.6 g
1 kg	=	2.205 lbs
1 metric ton	=	0.98 ton

Units of physical quantities

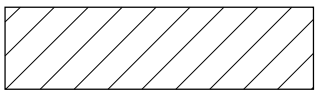


UNITS OF LENGTH

Units of length

Micron	1 µ	=	0.001 mm
Millimetre	1 mm	=	1000 µ
Centimetre	1 cm	=	10 mm
Decimetre	1 dm	=	10 cm
Metre	1 m	=	10 dm
Kilometre	1 km	=	1000 m
Inch	1"	=	25.4 mm
Foot	1’	=	0.305 m
Yard	1 Yd	=	0.914 m
Nautical mile	1 NM	=	1852 m
Geographical mile	1	=	1855.4 m

Units of area



UNITS OF AREA

Square millimetre	1 mm^2	
Square centimetre	1 cm^2	= 100 mm^2
Square decimetre	1 dm^2	= 100 cm^2
Square metre	1 m^2	= 100 dm^2
Are	1 a	= 100 m^2
Hectare	1 ha	= 100 a
Square kilometre	1 km^2	= 100 ha
Square inch	1 sq.in	= 6.45 cm^2
Square foot	1 sq.ft	= 0.093 m^2
Square yard	1 sq.yd	= 0.84 m^2
Square metre	1 m^2	= 10.76 ft^2
Acre	1	= 40.5 a

1 Acre = 100 cent

1 Cent = 436 Sq. ft.
1 Ground = 2400 Sq.ft.

1 Hectare = 2.47 acres

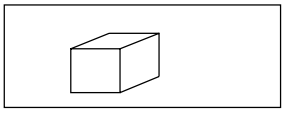
1 acre = 0.4047 Hectare

1 Hectare = 10000 sq. metre

Units of weight

UNITS OF WEIGHT

Milligram - force	1 mgf	
Gram-force	1 gf	1000 mgf
Kilogram-force	1 kgf	= 1000 gf
Tonne	1 t	= 1000 kgf
Ounce	1	= 28.35 gf
Pound	1 lbs	= 0.454 kgf
Long ton	1	= 1016 kgf
Short ton	1	= 907 kgf

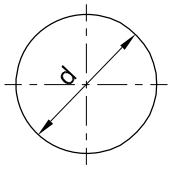


UNITS OF VOLUME AND CAPACITY

Units of volume and capacity

Cubic millimetre	1 mm^3	
Cubic centimetre	1 cm^3	= 1000 mm^3
Cubic decimetre	1 dm^3	= 1000 cm^3
Cubic metre	1 m^3	= 1000 dm^3
Litre	1 l	= 1 dm^3
Hectolitre	1 hl	= 100 l
Cubic inch	1 cu. in	= 16.387 cm^3
Cubic foot	1 cu. ft	= 28317 cm^3
Gallon (British)	1 gal	= 4.54 l
1cubic metre	1 m^3	= 1000 litres
1000 Cu.cm	1000 cm^3	= 1 litre
1 cubic foot	1 ft^3	= 6.25 Gallon
1 litre	1lt	= 0.22 Gallon

Circular unit



CIRCULAR OF UNIT

Radian

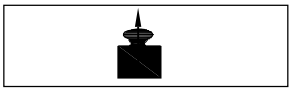
Relationship between Radian and Degree

1 Radian = $\frac{180^\circ}{\pi}$

180^0 = π Radian;

1 Degree = $\frac{\pi}{180}$ Radian

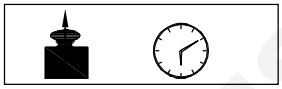
Work



WORK

Kilogram-force	1 kgfm	= 9.80665 J
Metre	1 kgfm	= 9.80665 Ws
Joule	1 J	= 1 Nm
Watt-second	1 Ws	= 0.102 kgfm
Kilowatt hour	1 kWh	= 3.6 x 10^6 J
		= 859.8456 $kcal_{IT}$
I.T.Kilocalorie	1 $kcal_{IT}$	= 426.kgfm

Power



POWER

Kilogram-force metre/second

1 kgfm/s	= 9.80665 W
Kilowatt	1 kW = 1000 W = 1000 J/s
	= 102 kgfm/s (approx.)
Metric horse power	1 HP = 75 kgfm/s
	= 0.736 kW
1 Calorie	= 4.187J

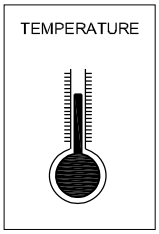
I.T.Kilocalorie/hour = 1 $kcal_{IT/h}$ = 1.163 W

Pressure

Pascal	1 Pa	= 1 N/m^2	1 atm	= 101325 Pa
Bar	1 bar = 10N/cm^2	= 100000 Pa–Torr	1 torr	= $\frac{101325}{760}$ ≈ 133.32 pa
Atmosphere	1 atm	= 1 kgf/cm^2	1 kgf/cm^2 = 735.6 mm of mercury	

TEMPERATURE

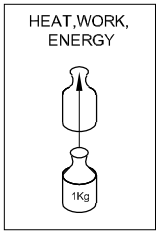
Scale	Freezing point	Boiling point
Centigrade (°C)	0°C	100°C
Fahrenheit (°F)	32°F	212°F
Kelvin (K)	273K	373K
Reaumur (°R)	0°R	80°R



$$\frac{°R}{80} = \frac{°C}{100} = \frac{K-273}{100} = \frac{°F-32}{180}$$

FORCE

Force	In C.G.S. System : Force (Dyne)	= Mass (gm)XAcceleration (cm/sec^2)
	In F.P.S. System : Force (Poundal)	= Mass (lb) X Acceleration (ft./sec^2)
	In M.K.S System : Force (Newton)	= Mass (Kg) x Acceleration (mtr./sec^2)
	1 Dyne	= 1 gm x1 cm/sec^2
	1 Poundal	= 1 lb x 1 ft/sec^2
	1 Newton	= 1 kg x 1 mtr/sec^2 = 10^5 dynes
	1gm weight	= 981 Dynes
	1 lb weight	= 32 Poundals
	1 kg weight	= 9.81 Newtons



ELECTRICAL QUANTITIES

V	Electric potential	V	Volt	V(W/A)
E	Electromotive force	V	Volt	V(W/A)
I	Electric current	A	Ampere	A
R	Electric resistance	Ω	Ohm	Ω (V/A)
e	Specific resistance	Ω m	Ohm metre	Vm/A
G	Conductance	Ω^{-1}	Siemens	S

Assignment - Answer the following question.

1 Convert 320 kilometres into miles _____

2 Convert 16 tons into kilograms _____

3 Convert 40 inches into centimetres _____

4 Convert 8 metres into feet _____

5 Convert 2.5 gallons into litres _____

6 Convert 5 litres into gallons _____

7 120°C = _____ °F.

8 Expand the abbreviations of the following

a N/m^2

b RPM

9 Convert the following S.I. units as required.

a Length

i 3.4 m = _____ mm

ii 10.2 km = _____ mile

b Mass

i 650 g = _____ kg

ii 120 mg = _____ g

c Force

i 1.2 N = _____ kg

ii 25 kg = _____ N

d Work, energy, amount of heat

i 120 KJ = _____ J

ii 300 wh = _____ kwh

e Power

i 0.2 kW = _____ W

ii 350 W = _____ kW

f Convert as required.

i 5 N = _____ KN

Unit, Fractions - Factors, HCF, LCM and problems

Prime Numbers and whole Numbers

Factor

A factor is a small number which divides exactly into a bigger number.e.g.

To find the factors of 24, 72, 100 numbers

24 = 2 x 2 x 2 x 3

72 = 2 x 2 x 2 x 3 x 3

100 = 2 x 2 x 5 x 5

The numbers 2,3,5 are called factors.

Definition of a prime factor

Prime factor is a number which divides a prime number into factors.e.g.

57 = 3 x 19

The numbers 3 and 19 are prime factors.

They are called as such, since 3 & 19 also belong to prime number category.

Definition of H.C.F

The Highest Common Factor

The H.C.F of a given group of numbers is the highest number which will exactly divide all the numbers of that group.e.g.

To find the H.C.F of the numbers 24, 72, 100

24 = 2 x 2 x 2 x 3

72 = 2 x 2 x 2 x 3 x 3

100 = 2 x 2 x 5 x 5

The factors common to all the three numbers are

2 x 2 = 4. So HCF = 4.

Definition of L.C.M

Lowest common multiple

The lowest common multiple of a group of numbers is the smallest number that will contain each number of the given group without a remainder.e.g.

- Factorise the following numbers

  7,17,20,66,128

  7,17 - These two belong to Prime numbers. Hence no factor except unity and itself.

2	20
2	10
	5

Factors of 20 = 2 x 2 x 5

2	66
3	33
	11

Factors of 66 = 2 x 3 x 11

2	128
2	64
2	32
2	16
2	8
2	4
	2

Factors of 128 = 2 x 2 x 2 x 2 x 2 x 2 x 2

- Select prime numbers from 3 to 29

  3,5,7,11,13,17,19,23,29

- Find the HCF of the following group of numbers HCF of 78, 128, 196

2	78
3	39
	13

78 = 2 x 3 x 13

2	128
2	64
2	32
2	16
2	8
2	4
	2

128 = 2 x 2 x 2 x 2 x 2 x 2 x 2

2	196
2	98
	49

196 = 2 x 2 x 49

HCF = 2

- Find LCM of 84,92,76

LCM =

2	84, 92, 76
2	42, 46, 38
3	21, 23, 19
	7, 23, 19

LCM = 2 x 2 x 3 x 7 x 23 x 19 = 36708

- To find out the LCM of 36, 108, 60

2	36, 108, 60
2	18, 54, 30
3	9, 27, 15
3	3, 9, 5
	1, 3, 5

LCM of the number

36, 108, 60 = 2 x 2 x 3 x 3 x 3 x 5 = 540

The necessity of finding LCM and HCF arises in subtraction and addition of fractions.

Unit, Fractions - Fractions - Addition, subtraction, multiplication & division

Description

A minimal quantity that is not a whole number. For e.g. .

$\frac{1}{5}$ a vulgur fraction consists of a numerator and denominator.

Numerator/Denominator

The number above the line in a vulgar fraction showing how many of the parts indicated by the denominator are taken is the numerator. The total number of parts into which the whole quantity is divided and written below the line in a vulgar fraction is the denominator. e.g.

$\frac{1}{4}, \frac{3}{4}, \frac{7}{12}$

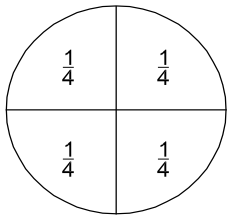
1,3,7 - numerators 4,12 - denominators

Fraction: Concept

Every number can be represented as a fraction.e.g.

$1\frac{1}{4} = \frac{5}{4}$, A full number can be represented as an apparent fraction.e.g. (Fig 1)

Fig 1



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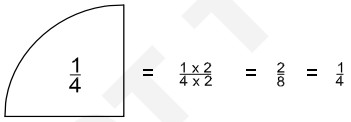
Fraction: Value

The value of a fraction remains the same if the numerator and denominator of the fraction are multiplied or divided by the same number.(Fig 2)

Fig 2



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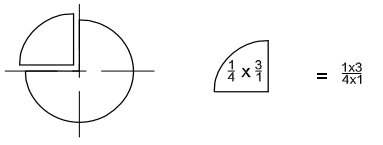
Multiplication

When fractions are to be multiplied, multiply all the numerators to get the numerator of the product and multiply all the denominators to form the denominator of the product. (Fig 3)

Fig 3



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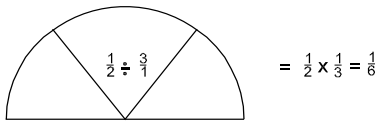
Division

When a fraction is divided by another fraction the dividend is multiplied by the reciprocal of the divisor. (Fig 4)

Fig 4



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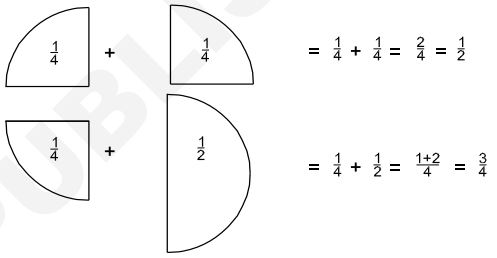
Addition and Subtraction

The denominators of the fractions should be the same when adding or subtracting the fractions. Unequal denominators must first be formed into a common denominator. It is the lowest common denominator and it is equal to the product of the most common prime numbers of the denominators of the fractions in question.(Fig 5)

Fig 5



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Examples

- Multiply $\frac{3}{4}$ by $\frac{2}{3}$,

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$$

- Divide $\frac{3}{8}$ by $\frac{3}{4}$,

$$\frac{3}{8} \div \frac{3}{4} = \frac{3}{8} \text{x} \frac{4}{3} = \frac{1}{2}$$

- Add $\frac{3}{4}$ and $\frac{2}{3}$,

$$\frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}$$

- sub $\frac{7}{16}$ from $\frac{17}{32}$

$$\frac{17}{32} - \frac{7}{16} = \frac{17}{32} - \frac{14}{32} = \frac{(17-14)}{32} = \frac{3}{32}$$

Types of fractions

- Proper fractions are less than unity. Improper fractions have their numerators greater than the denominators.
- A mixed number has a full number and a fraction.

Addition of fraction

$$\text{Add}\frac{1}{2}+\frac{1}{8}+\frac{5}{12}$$

To add these fractions we have to find out L.C.M of denominators 2,8,12.

Find L.C.M of 2,8,12

Step 1 L.C.M

2	2,8,12
2	1,4,6
	1,2,3

Factors are 2,2,2,3

Hence L.C.M = 2 x 2 x 2 x 3 = 24

Step 2

$$\frac{1}{2}+\frac{1}{8}+\frac{5}{12}=\frac{12}{24}+\frac{3}{24}+\frac{10}{24}$$

$$=\frac{12+3+10}{24}=\frac{25}{24}=1\frac{1}{24}.$$

Subtraction of fraction

$$\text{subtract } 9\frac{15}{32}\text{from}17\frac{9}{16}\text{or}(17\frac{9}{16}-9\frac{15}{32})$$

Step 1: Subtract whole number first 17 - 9 = 8

Step 2: L.C.M of 16,32 = 32

Since number 16 divides the number 32

Subtracting fractions = $\frac{3}{32}$

Adding with whole number from Step 1

$$\text{we get } 8+\frac{3}{32}=8\frac{3}{32}$$

Common fractions

Problems with plus and minus sign

Example

$$\text{solve } 3\frac{3}{4}+6\frac{7}{8}-4\frac{5}{16}-\frac{9}{32}$$

Rule to be followed

1 Add all whole numbers

2 add all + Numbers

3 Add all - Numbers

4 Find L.C.M of all denominators

Solution

Step 1: Add whole numbers = 3 + 6 - 4 = 5

Step 2: Add fractions = $\frac{3}{4}+\frac{7}{8}-\frac{5}{16}-\frac{9}{32}$

L.C.M of 4,8,16,32 is 32

$$\frac{24+28-10-9}{32}$$

$$=\frac{52-19}{32}$$

$$=\frac{33}{32}=1\frac{1}{32}$$

Step 3: Adding again with the whole number

$$\text{we get } 5+1\frac{3}{32}=6\frac{3}{32}$$

Examples

Common fractions

- Multiply

a $\frac{3}{8}\text{by}\frac{4}{7}=\frac{3}{8}\text{x}\frac{4}{7}=\frac{3}{14}$ b $\frac{2}{3}\text{x}\frac{3}{4}\text{x}\frac{5}{8}=\frac{5}{16}$

- Division

a $\frac{5}{16}\div\frac{5}{32}=\frac{5}{16}\text{x}\frac{32}{5}=2$

b $4\frac{2}{3}\div3\frac{1}{7}=\frac{14}{3}\div\frac{22}{7}=\frac{14}{3}\text{x}\frac{7}{22}=\frac{49}{33}=1\frac{16}{33}$

- Addition

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$$

$$\text{L..C.M } = 2,4,8 = 8$$

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=\frac{4+2+1}{8}=\frac{7}{8}$$

- Subtraction

$$5\frac{1}{4}-3\frac{3}{4}=5-3+\frac{1}{4}-\frac{3}{4}$$

$$=2+\frac{1}{4}-\frac{3}{4}=2\frac{1}{4}-\frac{3}{4}$$

$$=\frac{9}{4}-\frac{3}{4}=\frac{9-3}{4}$$

$$=\frac{6}{4}=\frac{3}{2}=1\frac{1}{2}$$

Assignment

1 Convert the following into improper fractions.

a $1\frac{2}{7} =$ _____

b $4\frac{3}{5} =$ _____

c $3\frac{3}{5} =$ _____

2 Convert the following into mixed numbers.

a $\frac{12}{11} =$ _____

b $\frac{36}{14} =$ _____

c $\frac{18}{10} =$ _____

3 Place the missing numbers.

a $\frac{11}{13} = \frac{x}{91}$ _____

b $\frac{3}{5} = \frac{42}{x}$ _____

c $\frac{9}{14} = \frac{x}{98}$ _____

4 Simplify.

a $\frac{45}{60} =$ _____

b $\frac{8}{12} =$ _____

5 Multiply.

a $5 \times \frac{2}{3} =$ _____

b $\frac{3}{4} \times 2 =$ _____

c $\frac{3}{4} \times \frac{5}{6} =$ _____

6 Divide

a $\frac{1}{4} \div \frac{3}{4} =$ _____

b $6 \div \frac{3}{4} =$ _____

c $\frac{3}{4} \div \frac{2}{7} =$ _____

7 Place the missing numbers.

a $\frac{2}{3} = \frac{1}{12} x$_____

b $\frac{14}{24} = \frac{1}{12} x$_____

c $\frac{7}{8} = \frac{1}{12} x$_____

8 Add the followings:

a $\frac{3}{4} + \frac{7}{12} =$ _____

b $\frac{7}{8} + \frac{3}{4} =$ _____

9 Subtract

a $\frac{4}{5} - \frac{2}{5} =$ _____

b $\frac{5}{6} - \frac{3}{4} =$ _____

10 Simplify

a $2\frac{6}{7} - \frac{3}{8} - \frac{1}{3} - 1\frac{1}{16} =$ _____

b $2\frac{2}{7} - \frac{5}{6} + 8 =$ _____

11 Express as improper fractions

a $5\frac{3}{4}$

b $3\frac{5}{64}$

c $1\frac{5}{12}$

Unit, Fractions - Decimal fractions - Addition, subtraction, multiplication & division

Description

Decimal fraction is a fraction whose denominator is 10 or powers of 10 or multiples of 10 (i.e.) 10, 100, 1000, 10000 etc. Meaning of a decimal number:-

12.3256 means

$$(1 \text{ X } 10) + (2 \text{ X } 1) + \frac{3}{10} + \frac{2}{100} + \frac{5}{1000} + \frac{6}{10000}$$

Representation

The denominator is omitted. A decimal point is placed at different positions of the number corresponding to the magnitude of the denominator

$$Ex. \frac{5}{10} = 0.5, \frac{35}{100} = 0.35 \frac{127}{10000} = 0.0127, \frac{3648}{1000} = 3.648$$

Addition and subtraction

Arrange the decimal fractions in a vertical order, placing the decimal point of each fraction to be added or subtracted, in succession one below the other, so that all the decimal points are arranged in a straight line. Add or subtract as you would do for a whole number and place the decimal point in the answer below the column of decimal points.

Decimal fractions less than 1 are written with a zero before the decimal point. Example: 45/100 = 0.45 (and not simply .45)

Add 0.375 + 3.686

$$\begin{array}{r} 0.375 \\ 3.686 \\ \hline 4.061 \end{array}$$

Subtract 18.72 from 22.61

$$\begin{array}{r} 22.61 \\ 18.72 \\ \hline 3.89 \end{array}$$

Multiplication

Ignore the decimal points and multiply as whole numbers. Find the total number of digits to the right of the decimal point. Insert the decimal point in the answer such that the number of digits to the right of the decimal point equals to the sum of the digits found to the right of the decimal points in the problem.

Multiply 2.5 by 1.25

= 25 x 125 = 3125. The sum of the figures to the right of decimal point is 3. Hence the answer is 3.125.

Division

Move the decimal point of the divisor to the right to make it a full number. Move the decimal point in the dividend to the same number of places, adding zeroes if necessary. Then divide.

Divide 0.75 by 0.25

$$0.25\overline{)0.75}$$

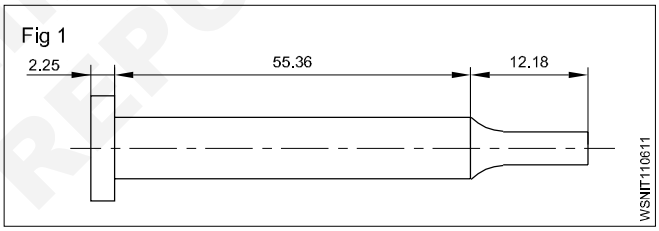
$$\frac{0.75}{0.25} \text{ x } \frac{100}{100} = \frac{75}{25}$$

$$25\overline{)75} = 3$$

Move the decimal point in the multiplicand to the right to one place if the multiplier is 10, and to two places if the multiplier is 100 and so on. When dividing by 10 move the decimal point one place to the left, and, if it is by 100, move them point by two places and so on.

Example

Allowance allowing 3 mm for cutting off each pin, how many pins can be made from a 900 mm long bar and how much material will be left out?



Total Length of pin = 2.25 + 55.36 + 12.18

= 69.79 mm

Cutting allowance = 3 mm

Total Length = length of pin + cutting allowance

= 69.79 mm + 3 mm

= 72.79 mm

Length of the bar = 900 mm

No.of pins to be cut = $\frac{900}{72.79}$ = 12.394

= 12 pins

Left out material = Total length - length of pin + cutting allowance

= 900 - 12 x 69.79 + 12 x 3

= 900 - 837.48 + 36

= 900 - 873.48

Left out length = 26.52 mm

Conversion of Decimals into fractions and vice-versa

- Convert decimal into fractions

Example

Convert 0.375 to a fraction

Now place 1 under the decimal point followed by as many zeros as there are numbers

$$0.375 = \frac{375}{1000} = \frac{15}{40} = \frac{3}{8}$$

$$0.375 = \frac{3}{8}$$

- Convert fraction into decimal

Example

- Convert $\frac{9}{16}$ to a decimal

Proceed to divide $\frac{9}{16}$ in the normal way of division but put zeros (as required) after the number 9 (Numerator)

$$\begin{array}{r} 0.5625 \\ 16\overline{)90000} \\ 80 \\ \hline 100 \\ 96 \\ \hline 40 \\ 32 \\ \hline 80 \\ 80 \\ \hline 0 \end{array}$$

$\frac{9}{16} = 0.5625$

Recurring decimals

While converting from fraction to decimals, some fractions can be divided exactly into a decimal. In some fractions the quotient will not stop. It will continue and keep recurring. These are called recurring decimals.

Examples

- convert $\frac{1}{3}, \frac{2}{3}, \frac{1}{7}$

a $\frac{1}{3} = \frac{10000}{3} = 0.3333 - \text{Recurring}$

b $\frac{2}{3} = \frac{20000}{3} = 0.666 - \text{Recurring}$

c $\left(\frac{1}{7} = \frac{10000}{7} = 0.142857142 - \text{Recurring}\right)$

Method of writing approximations in decimals

1.73556 = 1.7356 Correct to 4 decimal places
5.7343 = 5.734 Correct to 3 decimal places
0.9345 = 0.94 Correct to 2 decimal places

Multiplication and division by 10,100,1000

Multiplying decimals by 10

A decimal fraction can be multiplied by 10,100,1000 and so on by moving the decimal point to the right by as many places as there are zeros in the multiplier.

- 4.645 x 10 = 46.45 (one place)
- 4.645 x 100 = 464.5 (two places)
- 4.645 x 1000 = 4645 (three places)

Dividing decimals by 10

A decimal fraction can be divided by 10,100,1000 and so on, by moving the decimal point to the left by as many places as required in the divisor by putting zeros

Examples

- 3.732 ÷ 10 = 0.3732 (one place)
- 3.732 ÷ 100 = 0.03732 (two places)
- 3.732 ÷ 1000 = 0.003732 (three places)

Examples

- Rewrite the following number as a fraction

  453.273

  453.273

$$= (4 \times 100) + (5 \times 10) + (3 \times 1) + \frac{2}{10} + \frac{7}{100} + \frac{3}{100}$$

$$= 453\frac{273}{1000}$$

- Write the representation of decimal places in the given number 0.386

  3 - Ist decimal place
  8 - IInd decimal place
  6 - IIIrd decimal place

- Write approximations in the following decimals to 3 places.

  a 6.9453 ——> 6.945
  b 8.7456 ——> 8.746

- Convert fraction to decimal

$$\frac{21}{24} = \frac{7}{8} = 0.875$$

- Convert decimal to fraction

$$0.0625 = \frac{625}{10000} = \frac{5}{80} = \frac{1}{16}$$

Assignment

1 Write down the following decimal numbers in the expanded form.

a 514.726

b 902.524

2 Write the following decimal numbers from the expansion.

a $500 + 70 + 5 + \frac{3}{10} + \frac{2}{100} + \frac{9}{1000}$

b $200 + 9 + \frac{1}{10} + \frac{3}{100} + \frac{5}{1000}$

3 Convert the following decimals into fractions in the simplest form.

a 0.72

b 5.45

c 3.64

d 2.05

4 Convert the following fraction into decimals

a $\frac{3}{5}$

b $\frac{10}{4}$

c $24\ \frac{54}{1000}$

d $\frac{12}{25}$

e $\frac{8}{25}$

f $1\ \frac{3}{25}$

5 Addition of decimals

a 4.56 + 32.075 + 256.6245 + 15.0358

b 462.492 + 725.526 + 309.345 + 626.602

6 Subtract the following decimals

a 612.5200 – 9.6479

b 573.9246 – 215.6000

7 Add and subtract the following

a 56.725 + 48.258 – 32.564

b 16.45 + 124.56 + 62.7 - 3.243

8 Multiply the following

a By 10,100,1000

i 3.754 x 10

ii 8.964 x 100

iii 2.3786 x 1000

iv 0.005 x 1000

b By whole numbers

i 8.4 x 7

ii 56.72 x 8

c By another decimal figure (use calculator)

i 15.64 x 7.68

ii 2.642 x 1.562

9 Divide the following

a $\frac{62.5}{25}$

b $\frac{64.56}{10}$

c $\frac{0.42}{100}$

d $\frac{48.356}{1000}$

10 Division

a $\frac{16.8}{1.2}$

b $\frac{1.54}{1.1}$

11 Change the fraction into a decimal

i $1\ \frac{5}{8}$

ii $\frac{12}{25}$

12 Find the value

20.5 x 40 ÷ 10.25 + 18.50

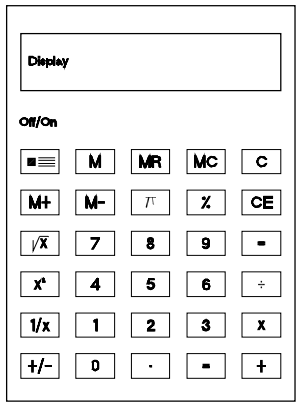
Unit, Fractions - Solving problems by using calculator

A pocket calculator allows to spend less time in doing tedious calculations. A simple pocket calculator enables to do the arithmetical calculations of addition, subtraction, multiplication and division, while a scientific type of calculator can be used for scientific and technical calculations also.

No special training is required to use a calculator. But it is suggested that a careful study of the operation manual of the type of the calculator is essential to become familiar with its capabilities. A calculator does not think and do. It is left to the operator to understand the problem, interpret the information and key it into the calculator correctly.

Constructional Details (Fig 1)

Fig 1



The key board is divided into five clear and easily recognizable areas and the display.

- **Data entry keys**

The entry keys are from [0] .....to [9]

and a key for the decimal point [.] .

- **Clearing keys**

These keys have the letter 'C'

[C] [CLR] Clear totally

[CE] Clear entry only

[CM] , [MC] Clear memory

[+] Addition key

[-] Subtraction key

[x] Multiplication key

[÷] Division key

[=] Equals key to display the result

- **Function keys**

[π] Pi key

[$\sqrt{x}$] Square root key

[%] Percentage key

[+/-] Sign change key

[x^2] Square key

[$\frac{1}{X}$] Reciprocal key

- **Memory keys**

[M] Store the display number

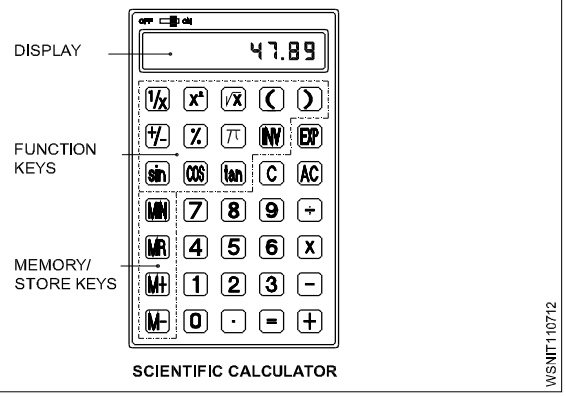
[M+] The displayed value is added to the memory

[M-] The displayed value is subtracted from the memory

[MR] [RCL] The stored value is recalled on to the display

Further functional keys included in Scientific calculators are as shown in Fig 2.

Fig 2



Sin Cos Tan ( ) For trigonometric functions and for brackets

Exp Exponent key

INV Some of the keys have coloured lettering above or below them. To use a function in coloured lettering, press INV key. INV will appear on the display. Then press the key that the coloured lettering identifies. INV will disappear from the display.

log , INV 10^x to obtain the logarithm of the displayed number and the antilogarithm of the displayed value.

INV R–P to convert displayed rectangular coordinates into polar coordinates.

INV P–R to convert displayed polar coordinates into rectangular coordinates.

- **The display**

The display shows the input data, interim results and answers to the calculations.

The arrangement of the areas can differ from one make to another. Keying in of the numbers is done via. an internationally agreed upon set of ten keys in the order that the numbers are written.

Rules and Examples:

- **Addition:** Example 18.2 + 5.7

Sequence	Input	Display
Input of the 1st term of the sum	1 8 . 2	18.2
Press + key	+	18.2
Input 2nd term of the sum. the first term goes into the register	5 . 7	5.7
Press the = key	=	23.9

- **Subtraction:** Example 128.8 - 92.9

Sequence	Input	Display
Enter the subtrahend	1 2 8 . 8	128.8
Press - key	-	128.8
Enter the minuend. The subtrahend goes into the register	9 2 . 9	92.9
Press the = key	=	35.9

- **Multiplication:** Example 0.47 x 2.47

Sequence	Input	Display
Enter multiplicand	. 4 7	0.47
Press x key	x	0.47
Enter multiplier, multiplicand goes to register	2 . 4 7	2.47
Press = key	=	1.1609

- **Division:** Example 18.5/2.5

Sequence	Input	Display
Enter the dividend	1 8 . 5	18.5
Press ÷ Key	÷	18.5
Enter the divisor goes to the register	2 . 5	2.5
Press = key	=	7.4

- **Multiplication & Division:**
  Example : 2.5 x 7.2 / 4.8 x 1.25

Sequence	Input	Display
Enter 2.5	2 . 5	2.5
Press x key	x	2.5
Enter 7.2	7 . 2	7.2
Press ÷ key	÷	18
Enter Open bracket	(	
Enter 4.8	4 . 8	4.8
Press x key	x	4.8
Enter 1.25	1 . 2 5	1.25
Enter Close bracket	)	6
Press = key	=	3.0

- Store in memory Example (2+6) (4+3)

Sequence	Input	Display
Workout for the first bracket	2	2
	+	2
	6	6
	=	8
Store the first result in	STO , M	8
x	or M+	
Workout for the 2nd bracket	4	4
	+	4
	3	3
	=	7
Press x key	x	7
Recall memory	RCL or MR	8
Press = key	=	56

- **Percentage:** Example 12% of 1500

Sequence	Input	Display
Enter 1500	1 5 0 0	1500
Press x key	x	1500
Enter 12	1 2	12
Press INV %	INV %	12
Press = key	=	180

- **Square root:** Example $\sqrt{2+\sqrt{3\times5}}$

Sequence	Input	Display
Enter 2	2	2
Press $\sqrt{a}$ key	$\sqrt{a}$	1.414
Press + key	+	1.414
Press bracket key	(	1.414
Enter 3	3	3
Press $\sqrt{a}$ key	$\sqrt{a}$	1.732
Press x key	x	1.732
Enter 5	5	5
Press $\sqrt{a}$ key	$\sqrt{a}$	2.236
Press bracket close key	)	3.873
Press = key	=	5.2871969
2 $\sqrt{}$ + (3 $\sqrt{}$ x 5 $\sqrt{}$) =		5.2871969

$\sqrt{2+\sqrt{3\times5}}$ = 5.287

- **Common logarithm:** Example log 1.23

Sequence	Input	Display
1 . 2 3 log	=	0.0899051

- **Power:** Example $123 + 30^2$

Sequence	Input	Display
1 2 3 + 3 0 INV X^2	=	1023

- **Before starting the calculations be sure to press the 'ON' key and confirm that '0' is shown on the display.**
- **Do not touch the inside portion of the calculator. Avoid hard knocks and unduly hard pressing of the keys.**
- **Maintain and use the calculator in between the two extreme temperatures of 0° and 40° C.**

- **Never use volatile fluids such as lacquer, thinner, benzine while cleaning the unit.**
- **Take special care not to damage the unit by bending or dropping.**
- **Do not carry the calculator in your hip pocket.**

Assignment

1 Using calculator solve the following

a 625 + 3467 + 20 + 341 + 6278 = _____

b 367.4 + 805 + 0.7 + 7.86 + 13.49 = _____

c 0.043 + 1.065 + 13.0 + 34.76 + 42.1 = _____

d 47160 + 1368.4 + 0.1 + 1.6901 + 134.267 = _____

2 Using calculator simplify the following

a 24367 - 4385 = _____

b 9.643 - 0.7983 = _____

c 4382.01 - 381.3401 = _____

d 693.42 - 0.0254 = _____

3 Using calculator find the values of the following

a 23 x 87 = _____

b 1376 x 0.81 = _____

c 678 x 243 = _____

d 0.75 x 0.24 = _____

4 Using calculator solve the following

a 22434 ÷ 3 = _____

b 4131 ÷ 243 = _____

c 469890 ÷ 230 = _____

d 3.026 ÷ 0.89 = _____

5 Solve the following

a $\frac{1170 \times 537.5}{13 \times 215}$ = _____

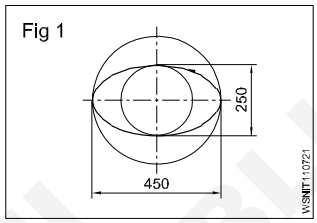
b $\frac{28.2 \times 18 \times 3500}{1000 \times 3 \times 0.8}$ = _____

6 Solve the following

a $\frac{(634 + 128) \times (384 - 0.52)}{8 \times 0.3}$ = _____

b $\frac{(389 - 12.2) \times (842 - 0.05 - 2.6)}{(3.89 - 0.021) \times (28.1 + 17.04)}$ = _____

7



2a = 450 mm(major axis)

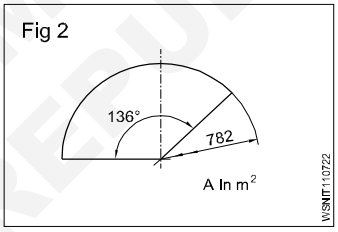
2b = 250mm(minor axis)

Perimeter of the ellipse

A = _____metre^2

Hint A = π x a x b unit^2

8



ø = 782 mm

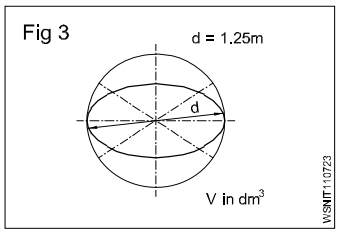
α = 136°

Area of the sector

A = _____

Hint $A = \frac{\pi \times d^2}{4} \times \frac{\alpha}{360°}$

9



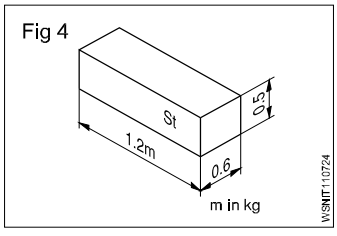
d = 1.25 metre

V = _____ dm^3

Volume of sphere

Hint $V = \frac{4}{3}\pi r^3$

10



L = 1.2 metres

B = 0.6 metre

H = 0.5 metre

'ρ' (rho) density of steel = 7.85 kg/dm^3

m = _____ kg

(mass 'm = V x ρ)

Square root, Ratio and Proportions, Percentage - Square and square root

a basic number
2 exponent

$\sqrt{\ }$ radial sign indicating the square root.

$\sqrt{a^2}$ square root of 'a' square

a² radicand

Square number

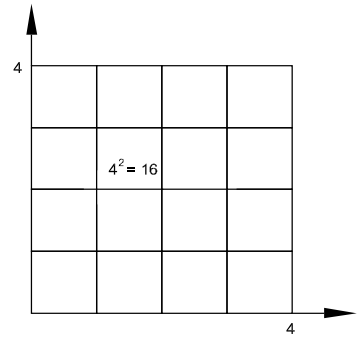
The square of a number is the number multiplied by itself.

Basic number x basic number = Square number

$a \times a = a^2$

$4 \times 4 = 4^2 = 16$

Fig 1



Splitting up

A square area can be split up into sub-areas. The largest square of 36 is made up of a large square 16, a small square 4 and two rectangles 8 each.

Large square 4 x 4 = 16 a^2

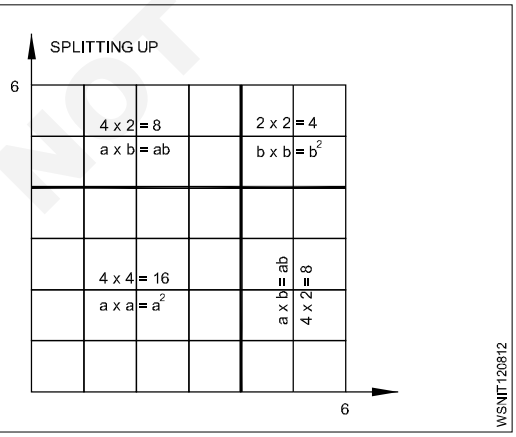
Two rectangles 2 x 4 x 2 = 16 2ab

Small square 2 x 2 = 4 b^2

Sum of sub-areas = 36 = $a^2 + 2ab + b^2$

$\sqrt{36} = \sqrt{a^2 + 2ab + b^2}$

Fig 2

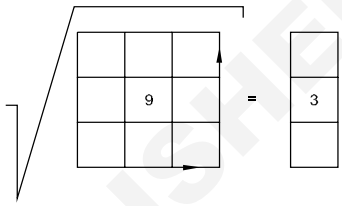


Result: In order to find the square root, we split up the square numbers.

Extracting the square root procedure

- Starting from the decimal point form groups of two figures towards right and left. Indicate by a prime symbol. $\sqrt{4624.00}$
- Find the root of the first group, calculate the difference, bring down the next group.
- Multiply the root by 2 and divide the partial radicand.
- Enter the number thus calculated in the divisor for the multiplication.

Fig 3



If there is a remainder, repeat the procedure.

		68
6		46,24
		36
128		1024
		1024
		0

$\sqrt{4624} = 68$

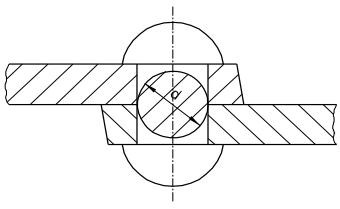
Basic number x basic number = Square

$\sqrt{\text{Square number}}$ = basic number

Example

The cross-section of a rivet is 3.46 cm². Calculate the diameter of the hole.

Fig 4



Rivet cross-section is the hole cross-section.

To find 'd',

Given that Area = 3.46 cm²

Area = 0.785 x d² (formula)

3.46 cm² = d² x 0.785

$$d^2 = \frac{3.46\ cm^2}{0.785}$$

$$d = \sqrt{\frac{3.46}{0.785}}\ cm$$

$d = 2.1\,cm\ (or)\ 21mm$

Square root, Ratio and Proportions, Percentage - Simple problems using calculator

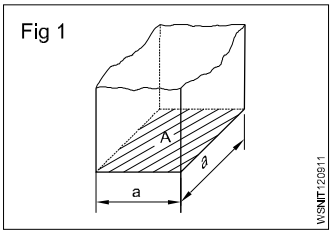
1 a $\sqrt{2916}$ = _____.

b $\sqrt{45796}$ = _____.

c $\sqrt{8.2944}$ = _____.

d $\sqrt{63.845}$ = _____.

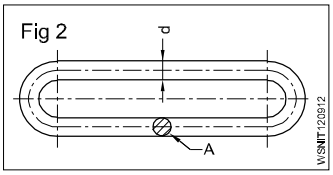
2



A = 2025 mm²

a = _____mm

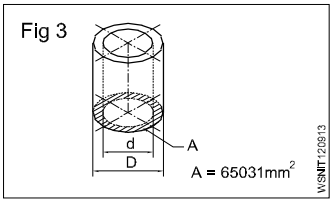
3



A = 176.715 mm²

d = _____mm

4

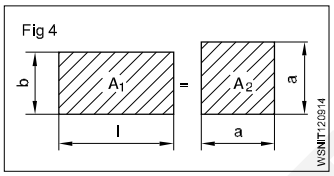


A = 65031 mm²

d = 140 mm

D = _____mm

5



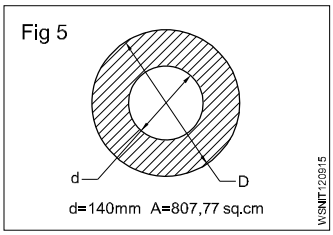
l = 58 cm

b = 45 cm

$A_1 = A_2$

a = _____cm

6

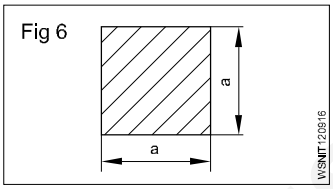


A = 807.77 cm²

d = 140 mm

D = _____mm

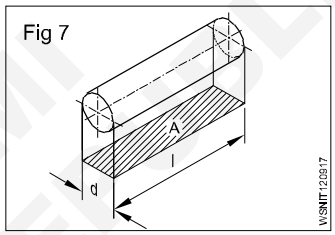
7



a x a = 543169 mm²

a = _____mm

8

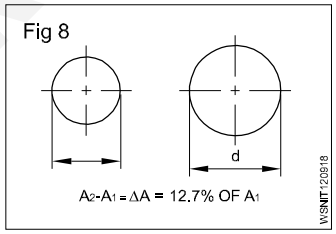


d : l = 1:1.5

A = 73.5 mm²

d = _____mm

9



increase in area

A = 12.7%

A = 360 mm²

d = _____mm

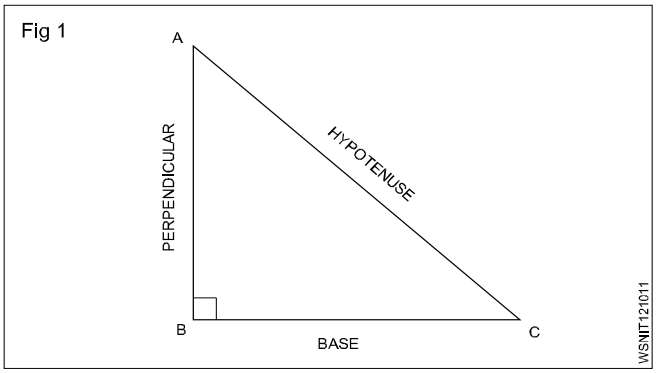
(d = diameter after the increase in area)

Square root, Ratio and Proportions, Percentage - Applications of pythagoras theorem and related problems

Applications of Pythagoras Theorem

Some of the applications of the Pythagoras theorem are; (Fig 1)

Fig 1



1 The Pythagoras theorem is commonly used to find the lengths of sides of a right-angled triangle.

2 It is used to find the length of the diagonal of a square.

3 Pythagoras theorem is used in trigonometry to find the trigonometric ratios like sin, cos, tan, cosec, sec and cot.

4 Pythagoras theorem is used in security cameras for face recognition.

5 Architects use the technique of the Pythagoras theorem for engineering and construction fields.

6 The Pythagoras theorem is applied in surveying the mountains.

7 It is also used in navigation to find the shortest route.

8 By using the Pythagoras theorem, we can derive the formula for base, perpendicular and hypotenuse.

9 Painters use ladders to paint on high buildings with the help of the Pythagoras theorem.

10 Pythagoras theorem is used to calculate the steepness of slopes of hills or mountains.

11 The converse of the Pythagoras theorem is used to check whether a triangle is a right triangle or not.

Application of pythagoras theorem in real life

Pythagoras theorem states that

"In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides".

1 The sides of this triangle have been named Perpendicular, Base and Hypotenuse.

2 The hypotenuse is the longest side, as it is opposite to the angle 90°.

3 The sides of a right triangle (say AB, BC and CA) which have positive integer values, when squared, are put into an equation, also called a Pythagorean triplet.

4 To calculate the length of staircase required to reach a window

5 To find the length of the longest item can be kept in your room.

6 To find the steepness of the hills or mountains.

7 To find the original height of a tree broken due to heavy rain and lying on itself

8 To determine heights and measurements in the construction sites.

Examples

1 What is the side AC if AB = 15 cm, BC = 25 cm.

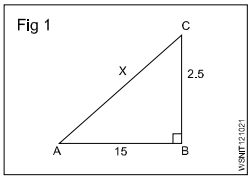
$AC^2 = AB^2 + BC^2$

$= 15^2 + 25^2$

$= 225 + 625 = 850$

$AC = \sqrt{850} = 29.155$ cm

Fig 1



2 What is the side BC if AB = 10 cm, AC = 30 cm.

$AC^2 = AB^2 + BC^2$

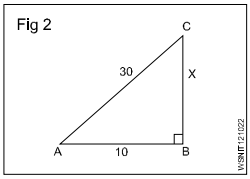
$30^2 = 10^2 + BC^2$

$900 = 100 + BC^2$

$BC^2 = 900 - 100 = 800$

$BC = 28.284$ cm

Fig 2



3 What is the side AB if BC = 20 cm, AC = 35 cm.

$AC^2 = AB^2 + BC^2$

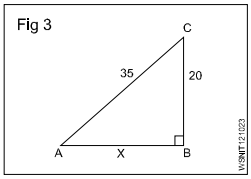
$35^2 = AB^2 + 20^2$

$1225 = AB + 400$

$AB^2 = 1225 - 400 = 825$

$AB = 28.72$ cm

Fig 3



4 What is the value of side BC if AB = 8 cm, AC = 24 cm.

$AC^2 = AB^2 + BC^2$

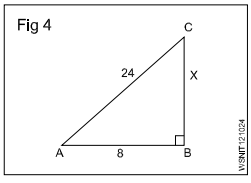
$24^2 = 8^2 + BC^2$

$576 = 64 + BC^2$

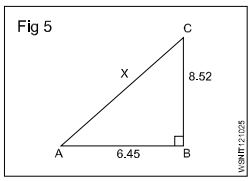
$BC^2 = 576 - 64 = 512$

$BC = \sqrt{572} = 22.63$ cm

Fig 4



5 What is the value side AC if AB = 6.45 cm, BC = 8.52 cm.



$AC^2 = AB^2 + BC^2$

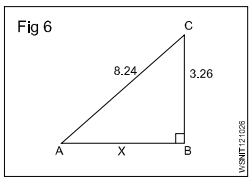
$AC^2 = 6.45^2 + 8.52^2$

$AC^2 = 41.60 + 72.59$

$= 114.19$

$AC = \sqrt{114.19} = 10.69$ cm

6 What is the value of side AB if BC = 3.26 cm, AC = 8.24 cm.



$AC^2 = AB^2 + BC^2$

$8.24^2 = AB^2 + 3.26^2$

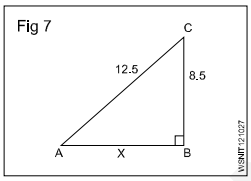
$67.9 = AB + 10.63$

$AB^2 = 67.9 - 10.63$

$= 57.27$

$AB = \sqrt{57.27} = 7.57$ cm

7 What is the value of side AB if AC = 12.5 cm, BC = 8.5 cm.



$AC^2 = AB^2 + BC^2$

$12.5^2 = AB^2 + 8.5^2$

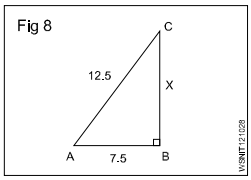
$156.25 = AB + 72.25$

$AB^2 = 156.25 - 72.25$

$= 84$

$AB = \sqrt{84} = 9.17$ cm

8 A ladder of 12.5 metre long is placed with upper end against a wall. The lower end being 7.5 metres from the wall. What height is the upper end above the ground.



$AC^2 = AB^2 + BC^2$

$BC^2 = AC^2 - AB^2$

$BC^2 = x^2$

$AC^2 = AB^2 + BC^2$

$12.5^2 = x^2 + 7.5^2$

$x^2 = (12.5)^2 - (7.5)^2$

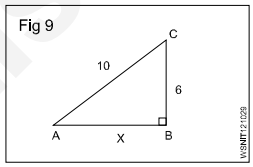
$= (12.5 + 7.5)(12.5 - 7.5)^2$

$= 20 \times 5$

$= \sqrt{100} = 10$

$x = 10$ m

9 What is the value of AB.



$AC^2 = AB^2 + BC^2$

$AB^2 = AC^2 - BC^2$

$AB^2 = x^2$

$AC^2 = AB^2 + BC^2$

$10^2 = x^2 + 6^2$

$x^2 = 10^2 - 6^2$

$= 100 - 36 = 64$

$x = \sqrt{64}$

$= 8$

$AB = 8$

Assignment

1 What is the value of side AB, in a right angled triangle of side AC = 10 cm and BC = 5 cm.

2 What is the value of side AC, in a right angled triangle of side AB = 6.5 cm and BC = 4.5 cm.

3 What is the value of side BC, in a right angled triangle of side AC = 14.5 cm and AB = 10.5 cm.

4 What is the value of side AC, in a right angled triangle of side AB = 7 cm and BC = 5 cm.

5 What is the value of side BC, in a right angled triangle of side AC = 13.25 cm and AB = 8.75 cm.

Square root, Ratio and Proportions, Percentage - Ratio and proportion

Ratio

Introduction

It is the relation between two quantities of the same kind and is expressed as a fraction.

Expression

a, b two quantities of the same kind. $\frac{a}{b}$ or a:b or a $\div$ b or a in b is the ratio.

Ratio is always reduced to the lowest terms.

Example

$$7:14 = \frac{7}{14} = \frac{1}{2} = 1:2$$

Proportion

It is the equality between the ratios, a : b is a ratio and c : d is another ratio. Both ratios are equal. Then

a :b :: c : d or $\frac{a}{b} = \frac{c}{d}$

Example

250 : 2000 :: 1 : 8

Proportion fundamentals

If $\frac{a}{b} = \frac{c}{d}$ then

- ad = bc
- $\frac{a}{c} = \frac{b}{d}$
- $\frac{b}{a} = \frac{d}{c}$
- $\frac{a+b}{b} = \frac{c+d}{c}$ and $\frac{a+b}{a} = \frac{c+d}{c}$
- $\frac{a-b}{b} = \frac{c-d}{d}$
- $\frac{a+b}{b+d} = \frac{a}{c} = \frac{c}{d}$

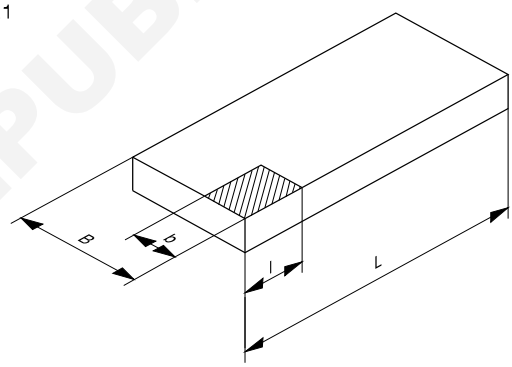
  3:4::6:8 or $\frac{3}{4} = \frac{6}{8}$
- 3 x 8 = 6 x 4
- $\frac{3}{6} = \frac{4}{8}$
- $\frac{4}{3} = \frac{8}{6}$
- $\frac{3+4}{4} = \frac{6+8}{8}$
- $\frac{3-4}{4} = \frac{6-8}{8}$
- $\frac{3+6}{4+8} = \frac{9}{12} = \frac{3}{4}$

Ratio - relation of two quantities of the same kind.
Proportion - equality between two ratios.

Example

- A steel plate of 800 x 1400 mm is to be drawn to a scale of 1:20. What will be the lengths in the Fig 1.

Fig 1



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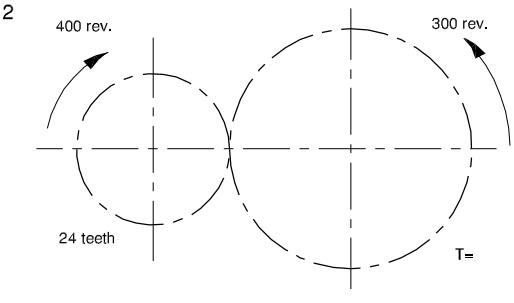
The reduction ratio is $\frac{1}{20}$.

B is reduced from 800 to 800 x $\frac{1}{20}$ = 40 mm.

L is reduced from 1400 x $\frac{1}{20}$ = 70 mm.

- Find the number of teeth of the larger gear in the gear transmission shown in the Fig 2.

Fig 2



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Speed ratio = 400 : 300

Teeth ratio = 24:T

$$\frac{400}{300}=\frac{T}{24}$$

$$\therefore T=\frac{24\times400}{300}=32\,\text{Teeth}$$

Find the ratio of A:B:C

If A:B= 2:3 and B:C=4:5

A:B = 2:3

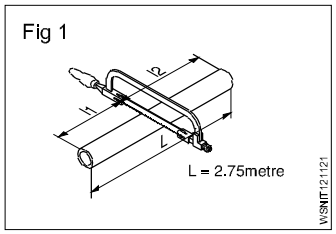
B:C = 4:5

A:B = 8 :12 (Ratio 2:3 multiply by 4)

B:C = 12:15 (Ratio 4:5 multiply by 3)

∴ A:B:C = 8:12:15

Assignment

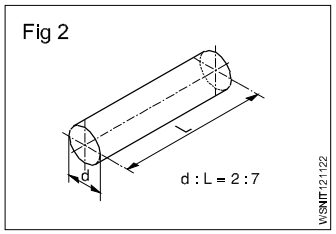
1 

$l_1 : l_2$ = 2:3

L = 2.75 metres

l_1 = _____ metres

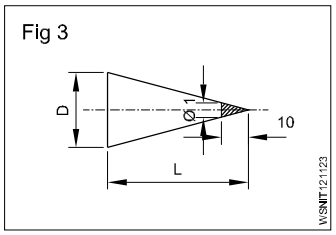
l_2 = _____ metres

2 

d: L of shaft = 2:7

d = 40 mm

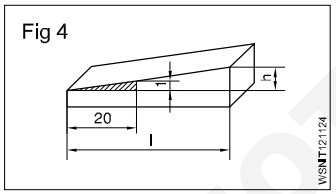
L = _____ mm

3 

D : L=1:10

L=150mm

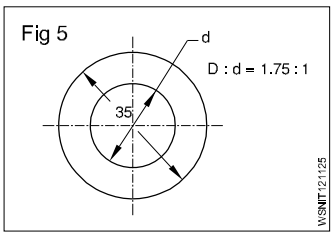
D=_____mm

4 

$$\frac{\Delta h}{l}=\frac{1}{20}$$

l = 140 mm

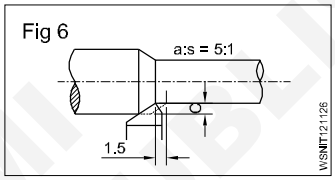
Δh = _____ mm

5 

D : d = 1.75: 1

D = 35 mm

d = _____ mm

6 

a:s = 5:1

s = 1.5mm

a =_____ mm

7 A:B=9:12

B:C=8:10

Then A:B:C=_____

8 A:B=5:6

B:C=3:4

Then A:B:C= _____

9 A:55=9:11

A = _____

10 15:9.3=40:x

x = _____

Square root, Ratio and Proportions, Percentage - Ratio and Proportion - Direct and indirect proportions

Proportion

Description

It is the equality between the ratios, a:b is a ratio and c:d is another ratio. Both ratios are equal. Then

a : b::c : d or e.g. 250 : 2000::1 : 8

Rule of three

A three step calculation

statement

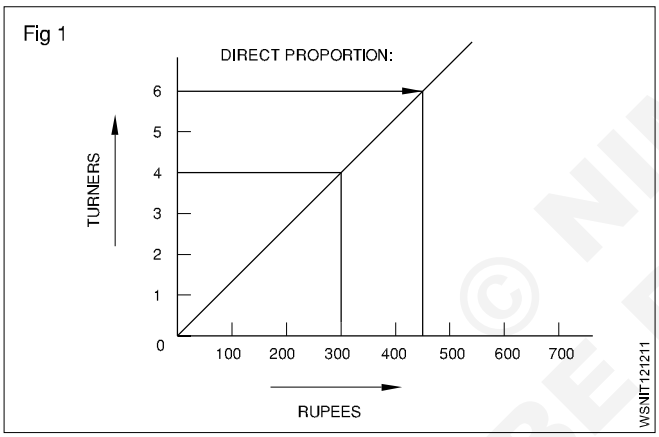
single

multiple.

Direct proportion

The more in one the more in the other - An increase in one denomination produces an increase in the other. (Fig 1)

Fig 1



Examples

1 4 turners earn 300 Rupees. How much will 6 Turners earn?

Statement

4 turners = 300 Rupees

Single

1 Turner = 75 Rupees

Multiple

6 Turners = 6 x 75 = 450 Rupees

2 One vehicle consumes 30 litres of petrol per day how much petrol is used by 6 Vehicles operating under similar condition.

One vehicle uses petrol = 30 litres per day.

Then six vehicles will use = 6 Times as much

= 6 x 30 = 180 litres/day.

3 4 vehicles consumes 120 gallons of petrol per day how much petrol will be used by 12 vehicles operating under the same condition.

4 vehicles use 120 gallons per day

1 Vehicle will use $\frac{120}{4}$ = 30 gallons/day

12 vehicles will use 12 x 30 = 360 gallons/day

Both examples are called simple proportion because only two quantities were used and the day is common for both ratios.

4 If 2 litres of petrol costs Rs 60. Find the cost of 50 litres.

Quantity of Petrol	Cost of Petrol
2 litres	Rs.60
50 litres	x

1 litre petrol = $\frac{60}{2}$ = Rs.30

50 litres petrol = 30 x 50 = Rs 1500

5 A 150mm dia gear meshes with 50mm dia gear. If the larger gear has 30 teeth. How many teeth will have the smaller gear have?

Gear dia	No. of Teeth
150 mm	30
50 mm	x

$$x = \frac{50}{150} \times 30 = 10 \text{ teeth.}$$

6 A mechanic assembles 7 machines in 2½ days. How long will it take time to assemble 70 machines at the same rate.

Machines	Days
7	2½
70	x

$$x = \frac{70 \times 2.5}{7} = 25 \text{ days}$$

Assemble for 70 machines will take 25 days.

7 A roll of wire weighs 1.24 kg from this roll a piece of 3.7cm long is cut and it is found to weigh 2.93 gm. What is the length of the wire in the roll?

Weight of wire	Length of wire
2.93 gm	3.7 cm
1.24 kg (1240 gm)	x

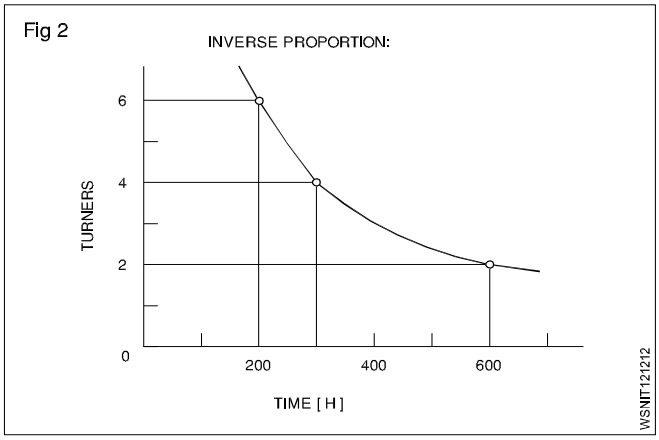
$$x = \frac{1240}{2.93} \times 3.7 = 1566 \text{ cm}$$

Length of wire = 1566 cm.

Indirect or inverse proportion

The more in one the lesser other - Increase in one quantity will produce a decrease in the other. (Fig 2)

Fig 2



Example

1 4 turners finish a job in 300 hours. How much time will 6 turners take to do the same job?

Solution procedure in three steps:

Statement 4 turners taken = 300 hours

The time will reduce if 6 turners to do the same job. Therefore this is inverse proportion.

Multiple fraction $\frac{4 \text{ Turners}}{6 \text{ Turners}} \times 300 \text{ hours}$

6 Turners = 200 hours

Result - The more the less.

2 8 workman take 6 days to complete a job. How many days it will take for 4 workman to complete the same job?

Workman	Days
8	6
4	x

$$x = \frac{8}{4} \times 6 = 12 \text{ days}$$

4 workers complete the work = 12 days.

3 5 men working on a job finished it in 32 days. Find out in how many days 8 men will finish the same job?

Men	Days
5	32
8	x

$$x = \frac{5 \times 32}{8} = 4 \times 5 = 20 \text{ days}$$

8 men will complete the job = 20 days.

4 An engine running at 150 rpm drives a shaft by pulley diameter is 55cm and that of the driven shaft pulley is 33 cm. Find the speed of the shaft?

Dia of pulley	Rpm of shaft
55 cm	150
33 cm	x

$$x = \frac{55 \times 150}{33} = 250 \text{ rpm.}$$

Speed of the 33cm diameter will run 250 rpm.

5 A pulley of 80 cm diameter is rotating at 100 rpm and drives another pulley of 40 cm diameter. Find the rpm of driven pulley. If slip is 2.5% find the rpm?

Dia of pulley	Rpm of pulley
80 cm	100
40 cm	x

40 cm diameter = 200 rpm.

Slip is 2.5% = 195 rpm.

Problems involving both

Example

2 turners need 3 days to produce 20 pieces. How long will it take for 6 turners to produce 30 such pieces?

Statement

2 turners, 20 pieces = 3 days

6 turners, 30 pieces = how many days.

First step (Fig 3)

Statement 2 turners for 20 pieces = 3 days

1 turner for 20 pieces = 3 x 2 = 6 days

Multiple 6 turners for 20 pieces = $\frac{6}{6}$ = 1 day

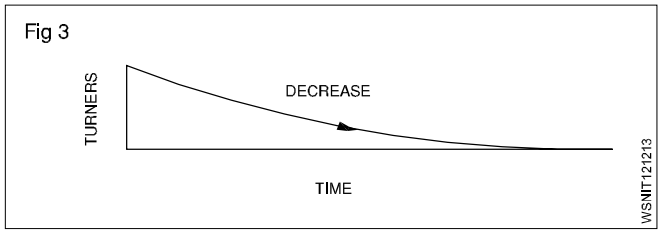
Statement 6 turners for 20 pieces = 1 day

Single 6 turners for 1 piece = $\frac{1}{20}$ days

Multiple 6 turners for 30 pieces = $\frac{1}{20} \times 30 = 1.5$ days

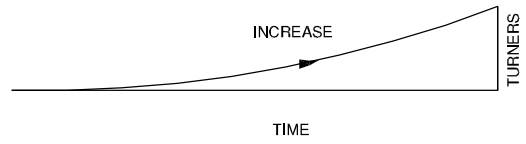
Inverse proportion - More the less.

Fig 3



Second step (Fig 4)

Fig 4



Direct proportion - More the more.

Solve the problem by first writing the statement and proceed to single and then to the multiple according to the type of proportion that is involved.

Introduction

Proportional fundamentals, as applicable to motor vehicle calculations are discussed below.

Simple Proportion

- Proportion

This is an equality between two ratios

Compound and Inverse proportions

- Compound proportions

Example

5 Fitter take 21 days to complete overhauling of 6 vehicles how long 7 Fitters will take to over haul 8 vehicles (Assume time of overhauling each vehicle is constant)

In this both direct and indirect proportions are used.

- 1 Fitter will overhauling 1 vehicle in days (shorter time).
- Quantities (No. of days) are taken in last as that is the answer required in this case.

Fitters	Vehicle	Days
5	6	21
7	8	x

$$\left(\frac{21\times5}{6\times7}\times8\right)=20 \text{ days}$$

Ans: 7 Fitters will overhaul 8 vehicles in 20 days.

Inverse proportion

Some times proportions are taken inversely.

Examples

- If one water pump fills the fuel tank in 12 minutes, two pumps will take half the time taken.

The time should not be doubled.

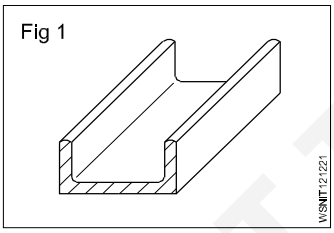
- 2 pumps will take 30 minutes to fill up a tank how long will 6 similar pumps take this to fill this tank.

Pump	Time
2	30
6	x

Ans: Time taken by 6 pumps = $\frac{30\times2}{6}=10$ minutes

Assignment

1 Fig 1

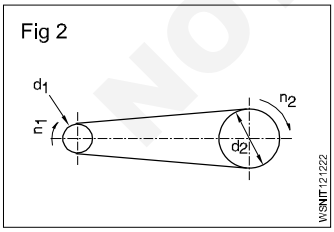


Length = 6.1 metre

Weight = 32 kgf

Weight of 1 metre of the same channel = _____ kgf

2 Fig 2



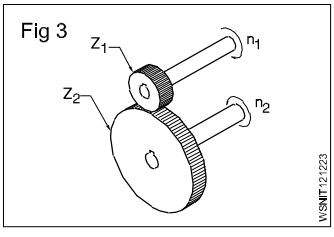
d_1 = 120 mm

d_2 = 720 mm

n_1 = 1200 rpm

n_2 = _____ rpm

3 Fig 3



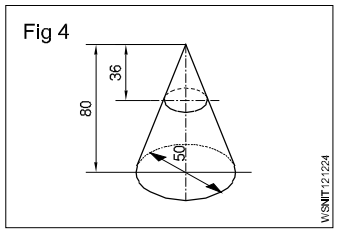
Z_1 = 42 T

n_2 = 96 rpm

n_1 = 224 rpm

Z_2 = _____ T

4 Fig 4



D = 50 mm

H = 80 mm

h = 36 mm

d = _____ mm

5 If a mechanic assembles 8 machines in 3 days, how long he will take to assemble 60 machines.

6 In an auto shop the grinding wheel makes 1000 rpm and the driven pulley is 200 mm dia. If the driving pulley is 150 mm dia. Find out the rpm of the driving pulley.

7 In a gearing of a vehicle the following facts are found.

A 180 mm dia of gear meshes with 60mm dia gear. If the bigger gear makes 60 rpm. What will be the rpm of smaller gear.

8 A vehicular job is completed by 5 mechanics in 4 days. If only 3 mechanics are available, in how many days the work can be completed.

Square root, Ratio and Proportions, Percentage - Percentage

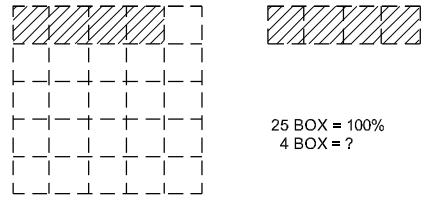
Percentage

Percentage is a kind of fraction whose denominator is always 100. The symbol for percent is %, written after the number. e.g. 16%.

$$\text{Ex. } \frac{16}{100} = 0.16$$

In decimal form, it is 0.16. Percentage calculation also involves rule of three. The statement (the given data), for unit, and then to multiple which is for calculating the answer. (Fig 1)

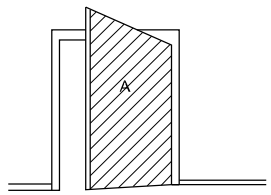
Fig 1



Example

The amount of total raw sheet metal to make a door was 3.6 metre^2 and wastage was 0.18 metre^2. Calculate the % of wastage. (Fig 2)

Fig 2



Solution procedure in three steps.

Statement:

Area of door (A) =3.6 m^2 = 100 %.

Wastage = 0.18 m^2

Single: $\frac{100}{3.6}$ x 1 m^2

Multiple: for 0.18 m^2= $\frac{100}{3.6}$ x 0.18.

Wastage = 5%.

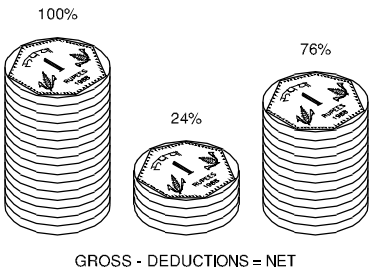
Analyse the given data and proceed to arrive at the answer through the unit.

Example

A fitter receives a take-home salary of 984.50 rupees.

If the deduction amounts to 24%, what is his total salary? (Fig 3)

Fig 3



Total pay 100%

Deduction 24%

Take home salary 76%

If the take home pay is Rs.76, his salary is 100.

For 1% it is $\frac{1}{76}$

For Rs.984.50, it is $\frac{1}{76}$ x 984.50.

For 100% it is $\frac{984.50}{76} X100 = 1295.39$

100% i.e. gross pay = Rs.1295.40.

Example 1

75 litres of oil is taken out from a oil barrel of 200 litres capacity. Find out the percentage taken in this.

Solution

% of oil taken = Oil taken out (litres) / Capacity of Barrel (litres) x 100

$$= \frac{75}{200} \text{ x } 100 = 37\tfrac{1}{2}\%$$

Example 2

A spare part is sold with 15%. Profit to a customer, to a price of Rs.15000/-. Find out the following (a) What is the purchase price (b) What is the profit.

Solution: CP = x,

CP = cost price

SP = sale price

SP=CP+15%of CP

$$15000 = x + \frac{15x}{100} = \frac{100x + 15x}{100}$$

$$x = \frac{1500000}{115} = 13043.47$$

Profit = SP-CP = 15000-13043.47 = 1956.53

Purchase price = Rs.13,043/,Profit = Rs. 1957

Example 3

Out of 80000 cars, which were tested on road, only 16000 cars had no fault. What is the percentage in this acceptance.

$$= \frac{160000}{80000} \times 100 = \frac{100}{5} = 20\%$$

Example 4

The price of a motor cycle dropped to 92% of original price and now sold at Rs.18000/- What was the original price.

Solution

Present price of Motor cycle Rs.18000

This is the value of 92% of original price

$$\text{Original Price} = 18000 \times \frac{100}{92} = \frac{1800000}{92}$$

= Rs.19565

Example 5

A Motor vehicle uses 100 litres of Petrol per day when travelling at 30 kmph. After top overhauling the consumption falls to 90 litres per day. Calculate percentage of saving.

Solution

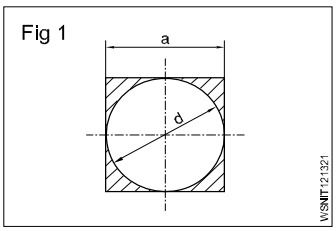
Percentage of saving = Decrease in consumption/Original consumption x 100

$$= (100 - 90)\frac{\text{litres}}{100} \times 100$$

$$= \frac{10}{100} \times 100$$

= 10% Saving in fuel.

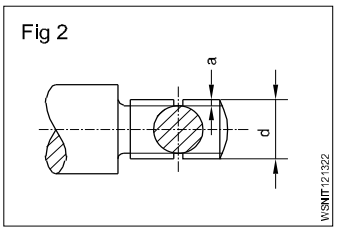
Assignment

1 

a = 400mm (side of square)

d = 400 mm

Wastage = _____ %.

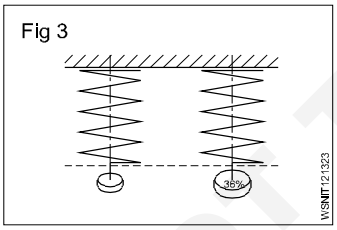
2 

d = 26mm

'a' depth of u/cut = 2.4mm

reduction of area at cross-section

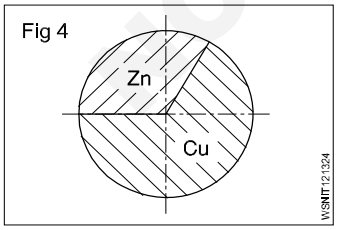
= _____ %

3 

Percentage of increase = 36%

Value of increase = 611.2 N/mm²

Original tensile strength = _____ N/mm².

4 

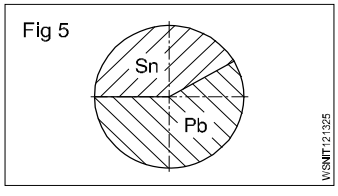
Copper in alloy = 27 kg

Zinc in alloy = 18 kg

% of Copper

= _____ %

% of Zinc = _____ %.

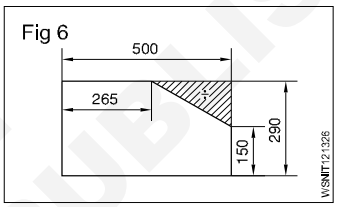
5 

Weight of alloy = 140 Kgf

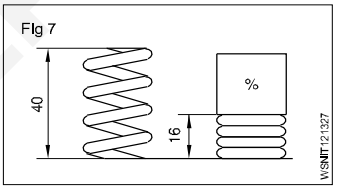
Weight of Sn 40%

Pb = _____ Kgf

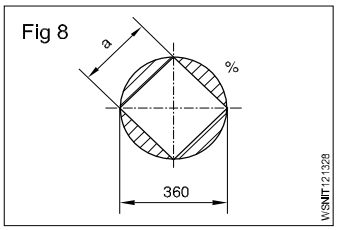
Sn = _____ Kgf.

6 

Shaded portion = _____ %.

7 

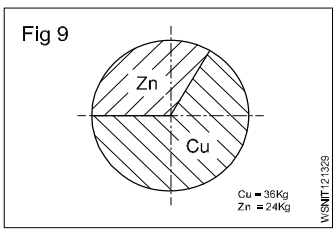
Compression length = _____ %.

8 

d = 360 mm

a = 0.707 x d

Wastage = _____ %.

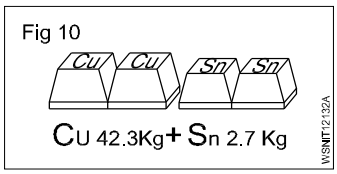
9 

Cu = 36 Kg

Zn = 24 Kg

Cu = _____ %

Zn = _____ %

10 

Cu = 42.3 Kg

Sn = 2.7 Kg

Cu _____ %

Sn = _____ %.

Square root, Ratio and Proportions, Percentage - Changing percentage to decimal and fraction

Conversion of Fraction into Percentage

1 Convert $\frac{1}{2}$ into percentage.

Solution: $\frac{1}{2}$ x 100

= 50%

2 Convert $\frac{1}{11}$ into percentage

Solution: $\frac{1}{11}$ x 100 = $\frac{100}{11}$

= 9.01%

Convert the following fraction into percentage.

1 $\frac{1}{4}$

2 $\frac{1}{5}$

3 $\frac{2}{3}$

4 $\frac{3}{8}$

Conversion of Percentage into Fraction

1 Convert 24% into fraction.

Solution: $\frac{24}{100} = \frac{6}{25}$

2 Convert $33\frac{1}{3}$% into fraction.

Solution: $\frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{100}{3} \times \frac{1}{100}$

$= \frac{1}{3}$

Convert the following percentage into fraction

1 15%

2 $87\frac{1}{2}$%

3 80%

4 12.5%

Conversion of Decimal Fraction into Percentage

1 Convert 0.35 into percentage.

Solution: 0.35 x 100

= 35%

2 Convert 0.375 into percentage.

Solution: 0.375 x 100

= 37.5%

Convert the following Decimal Fraction into Percentage

1 0.2

2 0.004

3 0.875

4 0.052

Conversion of Percentage into Decimal fraction

1 Convert 30% into decimal fraction.

Solution: $\frac{30}{100}$ = 0.3

2 Convert $33\frac{1}{3}$% into decimal fraction.

Solution: $\frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{100}{3} \times \frac{1}{100}$

$= \frac{1}{3}$ = 0.333

Convert the following percentage into decimal fraction

1 15%

2 7%

3 $12\frac{1}{2}$%

4 90%

Basic Electricity - Introduction and uses of electricity, molecule, atom, how electricity is produced, electric current AC,DC their comparison, voltage, resistance and their units

Electricity is a kind of energy. It is the most useful sources of energy which is not visible but its presence can be felt by its effects. Electricity is obtained by conversion of other forms of energy like heat energy, chemical energy, nuclear energy, mechanical energy and energy stored in water etc.,

To understand electricity, one must understand the structure of an atom.

Basically an atom contains electrons, protons and neutrons. The protons and neutrons are located in the centre of an atom and the electrons, a negative electric charge particle revolving around the nucleus in an atom. The proton has a positive charge. Neutrons are neutral and have no charge.

Sources of electricity

Battery

Battery stores electrical energy in the form of chemical energy and it gives power when required. Battery is used in automobiles and electronics, etc.,

Generator

It is a machine which converts the mechanical energy into electrical energy.

When a conductor rotates between a magnetic field using prime mover an emf will be induced. By using this method all types of AC and DC generator - generates power.

E.g. Thermal power station
Hydro power station
Nuclear power station
Wind power station
Solar power station

Thermo couple

If two dissimilar pieces of metals are twisted together and its joined end is heated in a flame, then a potential difference or voltage will be induced across the ends of the wires. Such a device is known as a Thermo couple. Thermo couple is used to measure very high temperature of furnaces.

Effects of electric current

When an electric current flows through a medium, its presence can be felt by its effects, which are given below.

1 Physical effect

Human body is a good conductor. when the body touches the bare current carrying conductor, current flows through the human body to earth and body gets severe shock or cause even death in many cases.

2 Magnetic effect

When an electric current passes through a coil, a magnetic field is produced around it.

E.g. : Electromagnet Motor, Generator, Electric bell

3 Chemical effect

When an electric current passes through an electrolyte, chemical action takes place. Because of that, an electrical energy is stored in a battery as a chemical energy.

E.g.: Electroplating, Cells and battery charging, refining of metals etc.,

4 Heating effect

When an electric current passes through any conductor, heat is produced in the conductor due to its resistance.

E.g. : Electric heater, Electric iron box, Electric lamp, Geyser, Soldering iron, Electric kettles, Electric welding etc.,

5 X-ray and Laser rays effect

When a high frequency voltage is passed through a vacuum tube, a special type of rays come out, which is not visible. These rays are called x-rays. Laser rays also can be produced by electric current.

6 Gas effect

When electrons pass through a certain type of sealed glass shell containing gas, then it emits light rays.

E.g: Mercury vapour lamp, Sodium vapour lamp, Fluorescent lamp, Neon lamp etc.,

Uses of Electricity

1	Lighting	- Lamps
2	Heating	- Heaters, ovens
3	Power	- Motor, fan
4	Traction	- Electromotive, lift, crane
5	Communication	- Telephone, telegraph, radio, wireless
6	Entertainment	- Cinema, radio, T.V.
7	Medical	- x-rays, shock treatment
8	Chemical	- Battery charging, electroplating
9	Magnetic	- Temporary magnets
10	Engineering	- Magnetic chucks, welding, x-rays of welding

Classification

- Static electricity
- Dynamic electricity

STATIC ELECTRICITY

If a dry glass rod is rubbed with silk cloth the glass rod gives out negative electrons, and therefore, becomes positively charged. The silk cloth receives negative electrons and therefore it becomes negatively charged. They acquire the property of attracting small pieces of paper etc. because like charges repel and unlike charges attract each other. The electric charge on the silk cloth is stationary and is called static electricity. This type of electricity cannot be transmitted from one place to another.

DYNAMIC ELECTRICITY

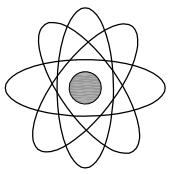
The electrons in motion are called current electricity or electric current. This type of electricity is carried through wires and cables. Therefore, this electricity can be transmitted from one place to another. This type of electricity can be produced by cells, batteries, generators alternators etc.

What is the difference between an atom and an element? How are molecules different from atoms? I am often asked these questions in my sessions over and over again and so I finally decided to write a comprehensive post on them. Find answers to all your questions in this section that is designed to help students explore and understand the relationship between atoms, elements, molecules, compounds and mixtures in a manner that is simple and easy to understand.

What is an Atom?

All the matter in the universe is made of tiny particles called atoms. There are 92 different kinds of atoms in nature. These 92 different atoms combine with one another to form different kinds of matter that we see in nature. (Fig 1)

Fig 1



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Gold, for example, is made of only gold atoms. When matter is made of only one kind of atom, it is called an element. In the same way, silver is another element which is made of only silver atoms. Because there are 92 different kinds of atoms in nature, there are 92 different kinds of elements. Other examples of an atom are K (potassium) and Fe (iron).

What is a Molecule?

A molecule is the smallest unit of a chemical compound and it exhibits the same chemical properties of that specific compound. As molecules are made up of atoms jointly held by chemical bonds, they can vary greatly in terms of complexity and size. The oxygen we breathe has a molecular formula O2. Should we consider this as an element or compound? When two or more atoms of the same elements combine together, we call them Molecules. So, we call O2 as an oxygen molecule. In the same way, we find hydrogen molecules H2, chlorine molecules Cl2 and others in nature.

Types of electric current

- Direct current
- Alternating current

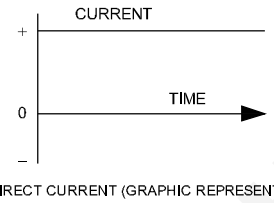
Direct current

In direct current (DC) the direction and magnitude of the current does not change (Fig 2). The steady current flow will be from the positive terminal to the negative terminal. (Fig 3)

Examples

DC Sources : Cells, batteries and DC generators (Fig 3)

Fig 2



DIRECT CURRENT (GRAPHIC REPRESENTATION)

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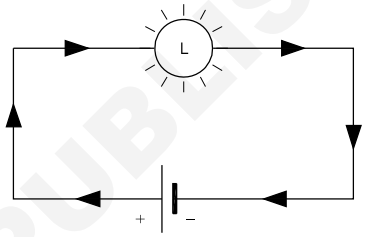
Fig 3



CURRENT FLOWS FROM +VE TERMIINAL TO –VE TERMINAL

DIRECT CURRENT (D.C)

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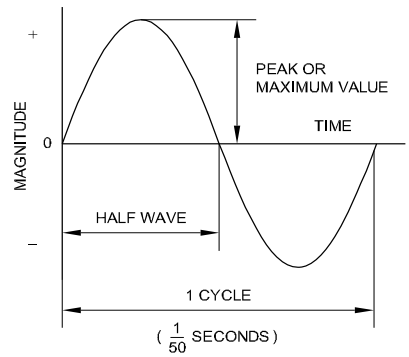
Alternating current (Fig 4)

The current flow will be from the phase terminal to the Neutral terminal. In the alternating current (AC) both the direction and magnitude of the current will be changing at definite intervals of time. The graph shows how an AC current or voltage changes with time. The current increases to the maximum value in one direction, falls to zero and increases to the maximum value in the other (opposite) direction before falling to zero again. Thus a cycle is one complete series of changes. The normal supply frequency is 50 cycles per second.

Fig 4



ALTERNATING CURRENT OR VOLTAGE

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Difference between AC and DC

	AC	DC
1	It is generated in the ranges of 6,600 V, 11000 V and 33,000 V.	It is generated up to 6,600 V only
2	Voltage can be stepped up or stepped down by using transformer	It is not possible
3	Transmission cost is less	Cost High
4	Less maintenance	High maintenance
5	Power up to 5,00,000 kw can be generated in a single alternator.	Power up to 10,000 kw can be generated in a single generator
6	AC generator can run at high speeds. So, speed control is not easy.	It can't run at high speeds. Speed control is easy.
7	Slip rings and brushes are used to collect the current.	Commutator and brushes are used to collect the current

Advantages of A.C.

i In transmission there is saving in copper wire.

ii Since there is no spark in A.C. machine there is no interference in Radio sound.

iii This can be produced to maximum voltage i.e. 33000volts.

iv Voltage can be dropped or raised with the help of transformers.

v Its mechanism is simple and cheap.

vi Output is more due to availability of more than one phase.

Disadvantages of A.C.:

i A single phase motor is not self-starter.

ii Due to thin wire in A.C., the voltage drop is more.

iii It cannot be used for electroplating and in charging secondary cells.

iv The speed of motors run by it is difficult to change.

v There is danger to security due to high voltage.

Electrical terms and units

Quantity of electricity

The strength of the current in any conductor is equal to the quantity of electrical charge that flows across any section of it in one second. If 'Q' is the charge and 't' is the time taken

then $I = \frac{Q}{t}$ $\quad Q = I \times t$

The SI unit of current is coulomb. Coulomb is equivalent to the charge contained in nearly 6.24×10^{18} electrons.

Coulomb

In an electric circuit if one Ampere of current passes in one second, then it is called one coulomb. It is also called ampere second (As). Its larger unit is ampere hour (AH)

1 AH = 3600 As (or) 3600 coulomb

Electro motive force (EMF)

It is the force which causes to flow the free electrons in any closed circuit due to difference in electrical pressure or potential. It is represented by 'E.' Its unit is Volt.

Potential difference (P.D)

This is the difference in electrical potential measured across two points of the circuit. Potential difference is always less than EMF. The supply voltage is called potential difference. It is represented by V.

Voltage

It is the electric potential between two lines or phase and neutral. Its unit is volt. Voltmeter is used to measure voltage and it is connected parallel between the supply terminals.

Volt

It is defined as when a current of 1 ampere flows through a resistance of 1 ohm, it is said to have potential difference of 1 volt.

Current

It is the flow of electrons in any conductor is called current. It is represented by 'I' and its unit is Ampere. Ammeter is used to measure the current by connecting series with the circuit.

Ampere

When 6.24×10^{18} electrons flow in one second across any cross section of any conductor, the current in it is one ampere.(or) If the potential difference across the two ends of a conductor is 1 volt and the resistance of conductor is 1 ohm then the current through is 1 ampere.

Resistance

It is the property of a substance to oppose to the flow of electric current through it, is called resistance. Symbol: R, Unit : Ohm (Ω), Ohm meter is used to measure the resistance.

Ohm

If the potential difference across the two ends of conductor is 1 volt and the current through it is 1 ampere, then the resistance of the conductor is 1 Ohm.

Laws of resistance

The resistance offered by conductor depends on the following factors.

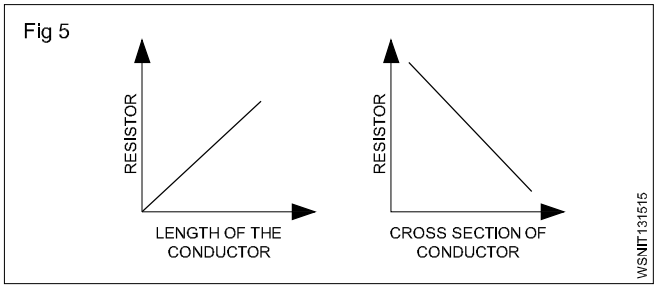
The resistance of the conductor

1 is directly proportional to the length of the conductor ($R \alpha L$)

2 Varies inversely proportional to its cross sectional area of the conductor $\left(R \alpha \frac{1}{A}\right)$

3 Depends on the material with which it is made.

Fig 5



4 depends on the temperature of the conductor

$$R \alpha L ; R \alpha \frac{1}{A}; R \alpha \frac{L}{A}; R = \rho \frac{L}{A}$$

Specific resistance

The specific resistance of a material is the resistance offered to a current it passed between the opposite faces of the unit cube of the material. Specific resistance is measured in Ohm - m or micro ohm - cm.

Each material has its own specific resistance or resistivity.

E.g. : Copper - 1.72 $\mu\Omega$ cm, Silver - 1.64 $\mu\Omega$ cm, Eureka - 38.5 $\mu\Omega$ cm, Iron - 9.8 $\mu\Omega$ cm,

Aluminium - 2.8 $\mu\Omega$ cm, Nickel - 7.8 $\mu\Omega$ cm.

$R = \frac{\rho l}{A}$ ohm cm

R = Resistance in ohms

l = Length of the conductor in cm

r = Specific Resistance in ohm cm

(symbol pronounced as rho)

A = Area of cross - section in cm^2

Basic Electricity - Conductor, insulator, types of connections - series and parallel

Conductors

Some materials and metals readily allow passage for electric current to flow. In such materials, called conductors, electrons are able to pass readily from atom to atom.

Properties of conductors

A good conductor should have the following properties.

Electrical properties

- The conductivity must be good.
- Electrical energy spent in the form of heat must be low.
- Resistivity must be low (to reduce voltage drop and loss).
- Increase in resistance with temperature must be low.

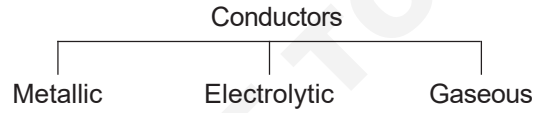
Mechanical properties

- Ductility (the property of being drawn into thin wires).
- Solderability: the joint should have minimum contact resistance.
- Resistance to corrosion: should not get rusted when used outdoors.
- Should withstand stress and strain.
- It should be easy to fabricate.

Economical factors

- Low cost.
- Easy availability.
- Easy to manufacture.

Classification of conductors



The best conductors are metallic. The commonly used conductors in electrical appliances and machines are described hereunder.

Silver

It is a soft and extremely malleable metal. Even though it is the best conductor, its use is limited because of its high cost.

Copper

It is a very good conductor. It is malleable and ductile, and also has high resistance to corrosion by liquids. Therefore, it is widely used for wires, cables, overhead conductors, bus bars and conducting parts of various electrical appliances.

Aluminium

It is a metal light in weight. It is also ductile, malleable and a good conductor of electricity. Nowadays, it is more widely used (since it is cheaper than copper) for wires and cables. All aluminium conductors (AAC) and aluminium conductors (steel reinforced) (ACSR) are used in overhead and transmission lines. (More details on copper and aluminium are furnished under the topic 'non-ferrous metals and alloys as applicable to electrical trades').

RESISTANCE WIRES

These are conductors with very high resistance for specific applications like filaments of incandescent lamps, heating elements etc. The following are a few examples:

1	Tungsten	2	Nichrome	3	Eureka
4	German silver	5	Manganin	6	Platinum
7	Mercury	8	Carbon	9	Brass.

The resistance values of the metallic resistances will increase with increase in temperature.

insulators

Description

These are the materials which offer very high resistance to the flow of current and make current flow very negligible or nil. These materials have very high resistance - usually of many megohms (1 megohm = 10^6 ohms) per centimetre cube. The insulators should also possesses high dielectric strength. This means that the insulating material should not break down or puncture even on application of a high voltage (or high electrical pressure) to a given thickness.

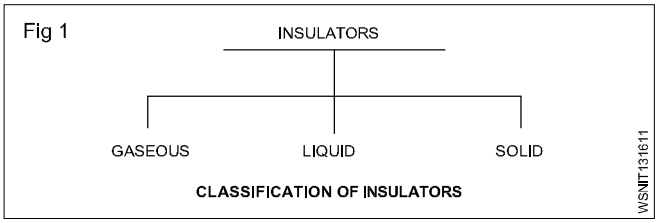
Properties of insulators

The main requirements of a good insulating material are:

- high specific resistance (many megohms/cm cube) to reduce the leakage currents to a negligible value
- good dielectric strength i.e. high value of breakdown voltage (expressed in kilovolts per mm)
- good mechanical strength, in tension or compression (It must resist the stresses set up during erection and under working conditions.)
- little deterioration with rise in temperature (The insulating properties should not change much with the rise in temperature i.e. when electrical machines are loaded.)
- non-absorption of moisture, when exposed to damp atmospheric condition. (The insulating properties, specially specific resistance and dielectric strength decrease considerably with the absorption of even a slight amount of moisture.)

Classification of insulators (Fig 1)

Fig 1



Air is an example of a gaseous insulator. Other examples are hydrogen, nitrogen and inert gases.

Liquid insulators

Mineral oils, synthetic liquids, resins and varnishes are the liquid insulators.

Transformer oil

In transformers the oil is used as an insulator and also for cooling of the transformer windings by convection. Therefore, the transformer oil should be dry and purified, since the presence of moisture will reduce the dielectric strength of the oil.

Purpose of transformer oil

- Transfer of heat by convection, from winding and core to the cooling surfaces.
- It maintains the insulation of winding and also extinguishes fire that occurs due to faults occurring in the windings.

Precaution

The insulating value of a transformer oil is reduced due to the formation of sludge as a result of oxidation due to air and temperature. To minimise oxidation, the oil should not be exposed to air.

Sludge is also formed due to the presence of acids and alkalis.

Sludge formation

- Reduces the rate of heat transfer.
- Blocks the ducts.
- Increases the operating temperature.

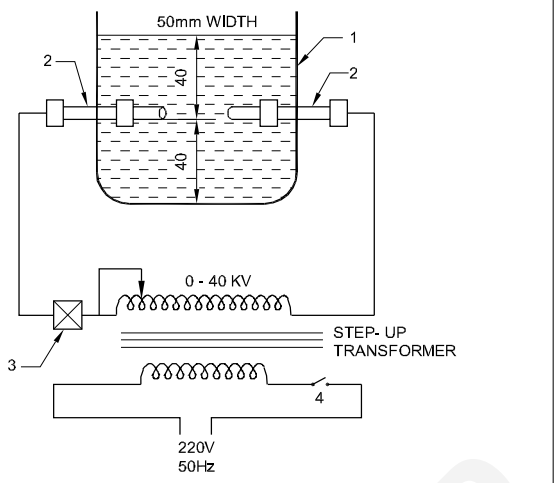
To prevent moisture from entering the oil, the whole apparatus is made airtight, and calcium chloride, silicagel fillets are used.

Testing of transformer oil as per ISI Standard (Fig 2)

Dielectrical strength test (Refer to Fig 2) : The oil should be 40 mm above and 40 mm below the electrodes. The gap between the two electrodes should be kept at 4 mm ± 0.02 mm).

A high voltage is applied across the electrodes through a step-up transformer, and increased till there is a spark in between the electrodes. The voltage noted on the voltmeters, when the spark occurs, is the breakdown voltage or dielectric strength of the oil. This is the maximum voltage the oil can withstand.

Fig 2



According to ISI specifications, the oil should be able to withstand 40 kV for one minute with a gap (4 mm ± 0.02 mm) between the electrodes and with the diameter of the electrodes as 13 mm.

Moisture test : In this test, an oil sample is cooled in a closed vessel down to 15-25°. A dry test tube, 12.5 mm in diameter and 125 mm long, is taken and an adequate quantity of oil is poured into it.

The tube containing the oil is heated rapidly with the help of an electric heater till the oil begins to boil. During the process, oil should not produce cracking.

The other tests are:

- acidity test
- sludge resistance test.

Electrical insulating varnishes

They are of two types

Oil and resin varnishes.

Solid insulators/insulating materials

Sl. No.	Classification	Examples
1	Mineral insulators	Mica, marble, slate.
2	Vitreous materials	Glass, quartz, procelain.
3	Rubber and rubber products	Rubber, vulcanised (India) rubber (V.I.R) ebonite
4	Waxes and compounds	Paraffin wax, bitumen.
5	Fibrous materials	Asbestos, paper, wood, Press pahn, leatheroid, cotton, silk, tapes etc.
6	Synthetic products	Bakelite, shellac, oil (for Transformer, Switchgear etc).

Paper

Various grades of insulating paper are available for use in capacitors, cables, etc. Paper, if moist, loses its insulating property. Therefore, it is used in an impregnated condition.

Wood

It is impregnated with oil or other substance for use as an insulator.

For example, in machine windings, bamboo wood is used as slot wedges.

Press board

It is widely used in windings to insulate parts which support windings. It is also used as spacers in electrical devices and transformers.

Asbestos

A fibrous, incombustible, fire- proof material - used for panel boards and as frames for winding resistance wires of regulators, rheostats etc.

Cotton

It is soaked in paraffin to avoid moisture. It is a good insulator for low voltages. It is used in conductors for armatures and field coils.

Silk

Like cotton, it is used for small jobs like telephone coils.

Tapes

Tapes of various types are used, such as cotton, silk, jute etc either pure or in impregnated form.

Empire cloth

It is made by varnishing a cotton cloth, silk or paper. It is not effected by moisture. It is available in yellow and black colours in different sizes. It is used as slots insulation in winding works and for coil insulation.

Press pahn

Press pahn is a form of paper made from hemp, rags, and wood pulp by special chemical treatment. It is widely used for lining armature slots, insulating coil sides, etc.

Leatheroid

It is a tough material made from cotton rags with chemical treatment. It is unaffected by grease or oil and is used for slot and coil insulation, transformer core coverings, etc.

Adhesive tape

It is used widely for taping of ends of conductors, leads and connections. Adhesive tape is made from cotton fabric coated with a compound of rubber, bitumen, resin, gum, etc. It dries when exposed to air. It is available in sizes $\frac{1"}{2}$, 3/4", 1" etc. These are also available as P.V.C. adhesive tape, cotton and bitumen tapes.

Bitumen

It is used for filling cable jointing boxes and for sealing the tops of the batteries etc. It is waterproof, but it will crack under certain conditions. It can be valcanised in the same manner as rubber.

Mica

It is a mineral and available as large slabs. It can be easily separated into thin sheets. It is fireproof, waterproof, and is a good insulator. It should be used carefully since it is liable to crack. It is used in heating elements of electric iron etc.

Marble and slate

Marble and slate are mechanically strong insulators and are non- hygroscopic. When polished they form good mountings for switchboards, switches, resistance frames, etc. Slate is used generally for low voltages.

Micanite

It is made by sticking together pieces of mica with insulating cement like shellac. It can be bent to any shape by heating and pressing. Therefore, it is used as insulator for slots of armatures and to insulate the commutator from the shaft.

Paraffin wax

It melts at 55°C and does not absorb water. It is used to impregnate paper, wood, pressboard etc to reduce their moisture absorption.

Bakelite

It can be moulded to any shape. It is heat-resistant and highly insulating. It will not absorb oil and moisture. It is used for bodies of switches, plugs, holders, regulators etc.

Rubber

It has high insulating properties. It is used mainly on lighting cables and for flexible cables. It deteriorates gradually when exposed to atmosphere. Rubber is being replaced now by elastic plastics such as PVC or polyethylene which can resist alkalis, acids and mineral oils.

Valcanised India Rubber (VIR)

This is manufactured by treating pure rubber with sulphur. It is stronger than pure rubber and is not affected much by change in temperatures. It is used as coverings for low and medium voltage wires and cables.

Ebonite or vulcanite

Ebonite or Vulcanite is vulcanised rubber containing about 30% to 50% of sulphur, and subjected to a prolonged heating at 150°C. The material is hard and can be moulded into different shapes. It is less affected by chemicals and moisture. It is used for making containers of lead acid batteries, cases for instruments and switchgears, terminal plates and low voltage panel boards etc. It should not be subjected to heat.

Shellac

It is a good varnish which is used to improve the insulation and moisture resisting properties of paper, cloth, wood, slate etc.

Enamel

By this, an insulation coating is given on winding wires.

Polychloroprene (PCP)

It is a plastic material used for insulation of cables. It is resistant to oil and petrol. It can be used in conditions of exposure to sulphur fumes, steam, ammonia, lactic acid and direct sunlight.

Glass

It is heat-resistant and suitable for high temperatures. It is used as insulators, envelopes for lamps, radio tubes etc.

Quartz

Quartz (Silica) is a good insulator. As it has a very low temperature coefficient of expansion, it does not crack with sudden variations in temperature. It is used for pyrometer sheaths, for heating elements, sparking plugs, etc.

Porcelain

Porcelain is not so brittle as glass and is very widely used for carrying bare conductors, for making fuse carriers and other electrical fittings.

Red fibre

Mainly used in motor and transformer winding work, for slot insulation, separators etc.

Insulators classified according to their temperature limits

The permissible temperature limit at which the insulators may be worked safely without deterioration, depends upon the type and class of the insulation as detailed below. (IS:1271/1958)

Class Y – maximum temperature 90°C

Cotton, silk, paper products, press board, wood, valcanised fibre - not impregnated or immersed in oil.

Class A – maximum temperature 105°C

Cotton, silk, paper products, wood, valcanised fibre when impregnated or immersed in liquid dielectric, varnished paper and wire enamel (class A).

Class E – maximum temperature 120°C

Wire enamel, cotton fabric and paper laminates treated with oil, modified asphalt and synthetic resins, varnished polyethylene, textile treated with suitable varnish.

Class B – Maximum temperature 130°C

Glass fibre, asbestos, varnished glass fibre, textile, varnished asbestos, built up mica treated with synthetic resin varnishes.

Class F – maximum temperature 155°C

Similar to class B materials but treated with silicone resins.

Class H – maximum temperature 180°C

Same as class F materials but treated with silicone resins of higher thermal stability than class F.

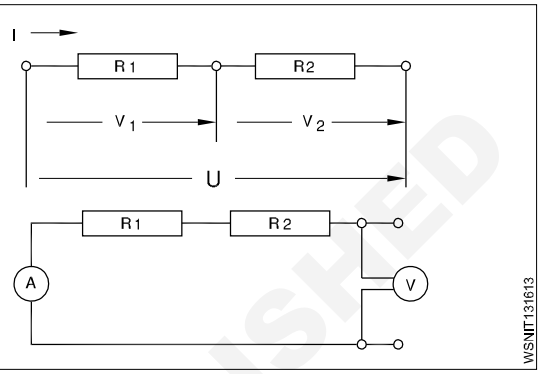
Class C – maximum temperature above 180°C

Mica, porcelain and other ceramics, glass, quartz, asbestos, treated glass fibre textile, treated asbestos, built up mica treated with silicone resins possessing superior thermal stability (limited stability up to 225°C).

Series Connection

The total resistance is equal to the sum of all the resistances. In a series connection the end of the first load is connected to the beginning of the second load and all loads are connected end to end. (Fig 3)

Fig 3



Features of series connection:

- The same current flows through all the loads.
- The voltage across each load is proportional to the resistance of the load.
- The sum of the voltages across each load is equal to the applied voltage.
- The Total resistance is equal to the sum of all the resistances.

$I = I_1 = I_2 = ...$

$V = V_1 + V_2 + ...$

$R = R_1 + R_2 + ...$

Example

Three resistances of 3 ohms, 9 ohms and 5 ohms are connected in series. Find their resultant resistance.

Solution

$R = R1 + R2 + R3$

$= 3\ \Omega + 9\ \Omega + 5\ \Omega$

Total resistance $= 17\ \Omega$

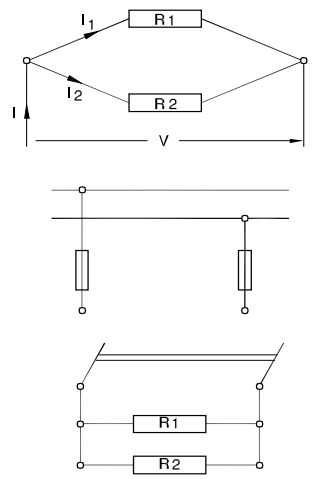
Parallel connection

In a parallel connection the beginning and the ends of the loads are connected together.

Features of parallel connection:

- The current flowing through each load depends upon the resistance of the load.
- The voltage across each load is the same and is equal to the voltage applied to the circuit.

Fig 4



- The total resistance of a parallel connection is always smaller than the smallest resistance in the circuit.
- In parallel connection the reciprocal of the total resistance is equal to the sum of the reciprocals of all resistances in the circuit.

$I = I_1 + I_2 + \ldots$

$V = V_1 = V_2 \ldots$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots\ldots$$

Example

Two resistances of 4 ohms and 6 ohms are connected in parallel. Determine the total resistance.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \quad \text{(since parallel connection)}$$

Therefore $\frac{1}{R} = \frac{1}{4} + \frac{1}{6} = \frac{10}{24} = \frac{5}{12}$

Therefore $R = \frac{24}{10}$ ohms = 2.4 ohms

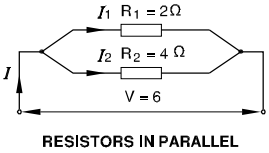
Example

Two resistors of 2 and 4 ohms are switched in parallel to a 6V battery

– Calculate the total resistance

– Find the total current and partial current.

Fig 5



Solution

Total resistance

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{1}{2} + \frac{1}{4} = \frac{2+1}{4}$$

$$= \frac{3}{4}\Omega$$

$$R_{tot} = \frac{4}{3} = 1\frac{1}{3}\Omega$$

I Total = $I_1 + I_2$ current

$$But\, I_1 = \frac{U}{R_1} = \frac{6V}{2\Omega} = 3A$$

$$I_2 = \frac{U}{R_2} = \frac{6V}{4\Omega} = 1.5A$$

I total = 3A + 1.5A

= 4.5 Amp

Basic Electricity - Ohm's law, relation between V.I.R & related problems

Ohm's law

V - Voltage in volts

I - Current in Ampere

R - Resistance in ohms.

In any closed circuit the basic parametres of electricity (Voltage, Current and resistance) are in a fixed relationship to each other.

Basic values

To clarify the basic electrical values, they can be compared to a water tap under pressure

Water pressure	- electron pressure	- Voltage
Amount of water	- electron flow	- Current
throttling of tap	- obstruction to electron flow	- Resistance

Fig 1

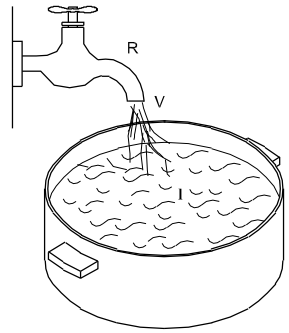


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Fig 2

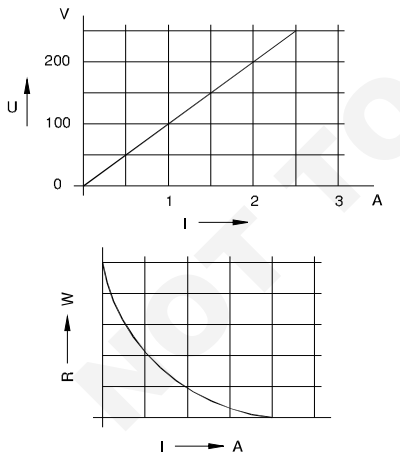

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Fig 3

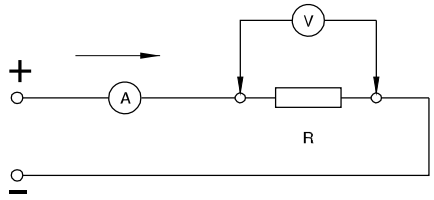

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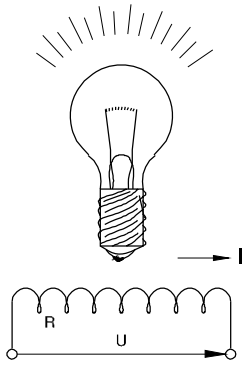
Fig 4



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Relationships

If the resistance is kept constant and the voltage is increased, the current is increased

$$I \propto V$$

If voltage is constant and the resistance is increased, current is decreased

$$I \propto \frac{1}{R}$$

Ohm's law

From the above two relationships we obtain Ohm's law,

$I = \frac{V}{R}$ which is conveniently written as V = R.I.

Ohm's law states that at constant temperature the current passing through a closed circuit is directly proportional to the potential difference, and inversely proportional to the resistance.

By Ohm's law $I = \frac{V}{R}$

EXAMPLE

A bulb takes a current of 0.2 amps at a voltage of 3.6 volts. Determine the resistance of the filament of the bulb to find R. Given that V = 3.6 V and *I* = 0.2 A.

To find 'R'. Given that V = 3.6V and I = 0.2 A

Therefore V = I x R

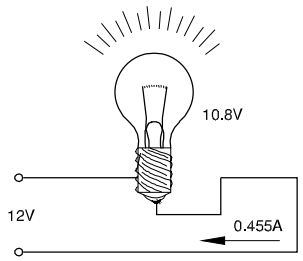
3.6 V = 0.2 A x R

Therefore $R = \frac{3.6V}{0.2A} = 18 ohms$

Example

The voltage supply to a filament lamp is 10.8V. The voltage should be 12V. Find out loss of voltage.(Fig 5)

Fig 5



Voltage drop = 12V – 10.8 = 1.2V

The supply voltage is called Potential difference.

Example

The Internal resistance of a dynamo is 0.1 ohm. The voltage of dynamo is 12V. What is the Voltage of dynamo when a current of 20 amps being supplied to an outside circuit.

Solution

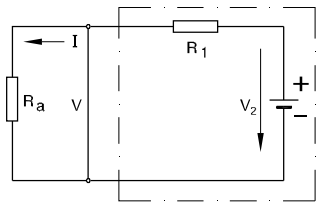
Voltage drop = Current x Internal resistance

= 20 x 0.1 volts

= 2 volts

Example (Fig 6)

Fig 6



The Internal resistance of a Battery is 2 ohms. When a resistance of 10 ohms is connected to a battery it draws 0.6 amps. What is the EMF of the battery.

P.D = Current flowing x Resistance

= 0.6 A x 10Ω

= 6 volts

V.D = Current flowing x Internal resistance of battery

= 0.6 x 2 volts

= 1.2 volts

EMF of the Battery = (6.00 + 1.2)V

= 7.2 volts

Resistance connections

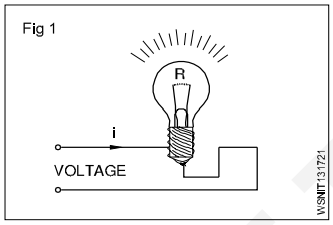
V - Voltage (in volts)

R - Resistance (in ohms)

I - Current intensity (in Amperes)

Assignment

1 Fig 1

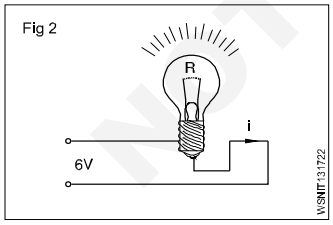


R = 40 Ohms

I = 6.5 Amps

V = _____ Volts

2 Fig 2

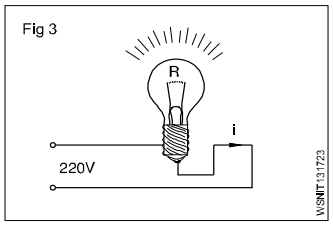


V = 6 Volts

I = 0.5 Amps

R = _____ Ohms

3 Fig 3

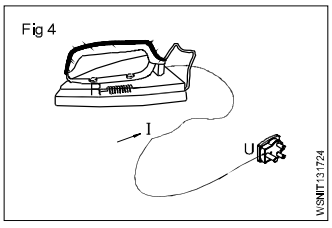


V = 220 Volts

R = 820 Ohms

I = _____ Amps

4 Fig 4

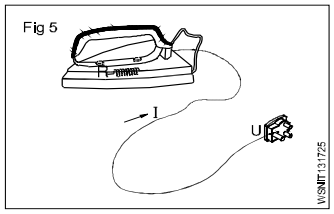


I = 4.5 Amps

V = 220 Volts

R = _____ Ohms

5 Fig 5

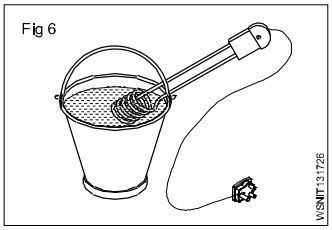


R = 50 Ohms

V = 220 Volts

I = _____ Amps

6 Fig 6



V = 110 Volts

I = 4.55 Amps

R = _____ Ohms

Basic Electricity - Electrical power, energy and their units, calculation with assignments

Electric Power

In mechanical terms we defined power as the rate of doing work. The unit of power is Watt. In an electrical circuit also the unit of electrical power is 1 Watt. In mechanical terms 1 Watt is the work done by a force of 1 N to move the body through 1 metre in one second. In an electrical circuit, the electromotive force overcomes the resistance and does work. The rate of doing work depends upon the current flowing in the circuit in amperes. When an e.m.f of one volt causes a current of 1 ampere to flow the power is 1 Watt.

Hence Power = Voltage x Current

$$P = V \times I$$

Power in Watts = Voltage in Volts x Current in Amperes

Electric work, energy

Electrical work or energy is the product of electrical power and time

Work in Watt seconds = Power in Watts x time in sec

$$W = P \times t$$

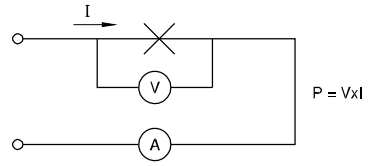
Since 1 joule represents 1 Watt x 1 sec, which is very small, larger units such as 1 Watt hour and 1 kilowatt hour are used.

1 W.h = 3600 Watt sec.

1 Kwh = 1000 Wh = 3600000 Watt sec

Note: The charge for electric consumption is the energy cost per Kwh and it varies according to the country and states.

Fig 1



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V - Voltage (Volts) V

i - Current Intensity (Amperes) A

P - Power (Watts, Kilowatts) W, kW

W - Work, Energy (Watt hour, Kilowatt hour) wh, Kwh

t - time (hours) h

Fig 2

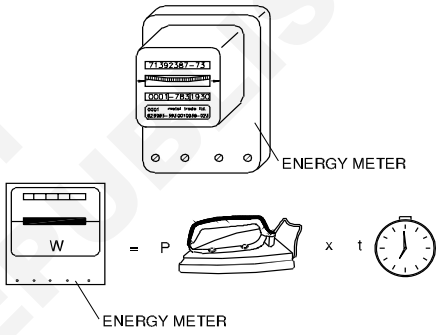

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Table of analogies between mechanical and electrical quantities

Mechanical quantity	Unit	Electrical quantity	Unit
Force 'F'	N	Voltage 'V'	V
Velocity $v = \frac{\text{Displacement}}{\text{Time}}$	m/s	Current *I*	A
Time *t*	seconds	Time *t*	seconds
Power $P = F \times v$	N $\frac{m}{sec}$	Power $P = V \times i$	$W = V \times A$
Energy = $F \times v \times t$	j = Nm	Energy $W = V \times i \times t$	$j = W \times s$

$$W = V I = I^2R = \frac{V^2}{R}$$

$$R = \frac{V}{I} = \frac{V^2}{W} = \frac{W}{I^2}$$

$$V = IR = \frac{W}{I} = \sqrt{WR}$$

$$I = \frac{V}{R} = \frac{W}{V} = \sqrt{\frac{W}{R}}$$

Example

1 Calculate the power rating of the lamp in the circuit, if 0.25 amperes of current flows and the voltage is 240 volts.

P = V x I

V = 240 Volts

I = 0.25 Amperes

Therefore Power = 240 Volts x 0.25 Amperes

= 60 Volts Ampere

But 1 Watt = 1 Volt x 1 Ampere

Therefore Power = 60 Watts

2 A current of 15 amperes flow through a resistance of 10 Ohms. Calculate the power in kilowatts consumed.

Given that R = 10 and *I* = 15A

Power = V x *I* = *I* x R x *I* = I^2 x R

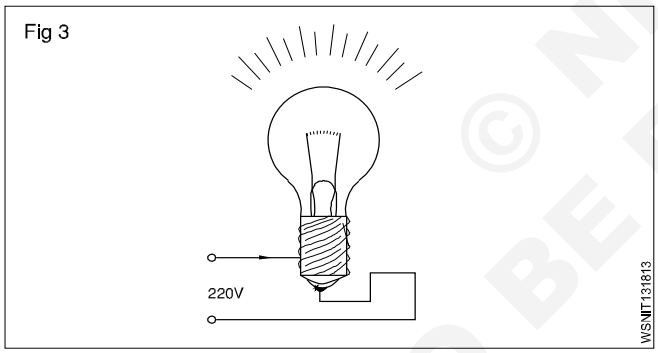
Therefore Power = 15^2 x 10 = 2250 Watts = 2.25 kW

3 At a line voltage of 200 Volts a bulb consumes a current of 0.91 amperes. If the bulb is on for 12 hour calculate the work in Wh to find the work given that V = 200 Volts.

I = 0.91 Amps.

t = 12 hours

Fig 3



Therefore Power=V x *I* = 200 Volts x 0.91 Amps

= 182 Watts

Therefore Work = P x *t* = 182 Watts x 12 hours

= 2184 Watt hour.

4 An adjustable resistor bears the following label: 1.5 k Ohms/0.08 A. What is its rated power?

Given: R = 1.5 k Ohms; I = 0.08 A

Find: P

V = R x I = 1500 Ohms.0.08 A = 120 volts

P = V x I = 120 volts.0.08 A = 9.6 W alternatively: P = 1^2.R = $(0.08\ A)^2$.1500 Ohms = 9.6 W.

5 Find the current and power consumed by an electric iron having 110Ω resistance when feed from a 220 v supply

Resistance of electric iron (R) = 110 ohms

Voltage (V) = 220 volts

Current (I) = $\frac{V}{R}$

= $\frac{220}{110}$ 2 amperes

Power (w) = V x I

= 220 x 2

= 440 watts

6 Find the total power if four 1000 W, 180 volt heaters are connected in series across 240 V supply and current carrying capacity is 15 amp. Find the total power.

Connection = Series

No. of heaters = 4

Heater power (W) = 1000 watts

Heater voltage = 180 V

Supply voltage = 240 V

Heater resistance (R) = $\frac{V^2}{W}$

= $\frac{180 \times 180}{1000} = \frac{324}{10}$

= 32.4 ohms

Total resistance = 32.4 x 4 = 129.6 ohms

Total current (I) = $\frac{V}{R}$

= $\frac{240}{129.6}$ = 1.85 amperes

Total Power (W) = V x I

= 240 x 1.85 = 444 watts

7 If a 40 watt fluorescent lamp draws a current of 0.10 ampere. How much voltage will be required to illuminate it?

Lamp power (W) = 40 watt

Current (I) = 0.10 ampere

Voltage (V) = $\frac{W}{I}$

= $\frac{40}{0.1}$ = 400 volts

8 Find the cost of running 15 HP motor for 15 days @ 6 hrs per day and the cost of energy is Rs. 3 per unit.

Motor power (w) = 15 HP

= 15 x 746 = 11,190 watts

Consumption per day = 11,190 x 6

= 67140 = 67.14 KWH

Consumption for 15 days = 67.14 x 15

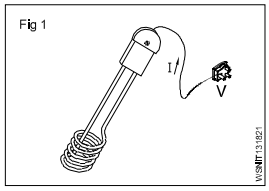
= 1007 KWH (or) unit

Cost per unit = Rs. 3

Cost for total energy = 3 x 1007 = Rs. 3021

Assignment

1



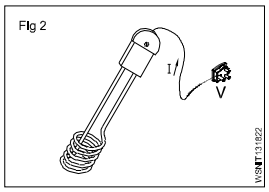
Current Consumed

I = 0.136 A

Voltage 'V' = 220 V

P = _____ Watts

2

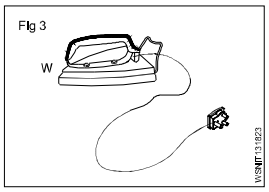


P = 500 Watts

I = 2.27 A

V = _____ v

3

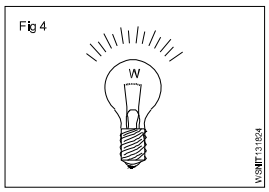


P = 750 W

V = 220 v

I = _____ A

4

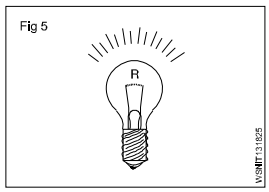


P = 60 W

V = 200 v

R = _____ W

5

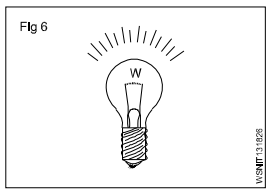


I = 0.455 A

R = 484 ohms

P = _____ Watts

6

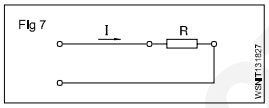


P = 550 W

R = 22 ohms

I = _____ A

7

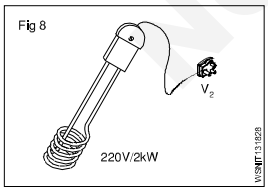


P consumed = 1.8 kW

R = 8 ohms

V = _____ v

8



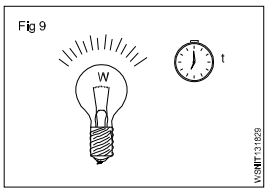
I consumed = ____ A

P = 2 kW

V_1 = 220V (Heating element voltage)

R = _____ W

9



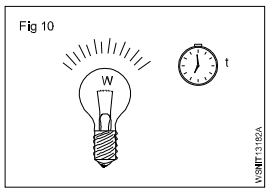
P = 100 W

t = 1 hour

Energy consumption

= _____ kWh

10



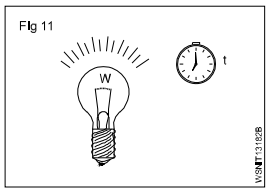
Energy consumed

'W' = 1 kWh

Power 'P' = 100 W

t = _____ hr

11

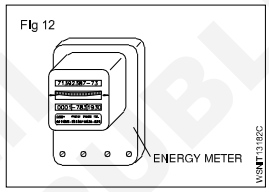


W = 1.5 kWh

t = 45 min.

P = _____ kW.

12



Energy metre reading

W_1 = 6755.3 kWh

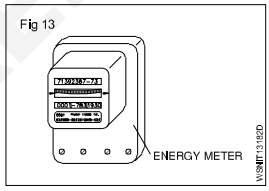
Increases to W_2

= 6759.8 kWh

t = 45 min.

P = _____ kW.

13



Power consumed

'P' = 6.2 kW

t = 8 hours

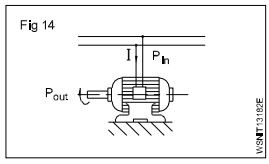
Charge per kwh

= 1.25 Rupees

Total cost

= _____ Rupees

14



I = 5.45 A

V = 220 v

Energy consumed

= 1 kWh

t = _____ hr.

Trigonometry - Measurement of angles

Introduction:

Trigonometry is the branch of mathematics which deals with the study of measurement and relationship of the three sides and three angles of a triangle.

Units:

Measurement of Angles

There are three systems of measuring the angle:

(i) Sexagesimal System

This is called British System. In this system, one right angle is divided into 90 equal parts which are called degrees.Each part is divided into 60 parts which are called minutes.Each minute is divided into 60 parts which are called seconds.The parts so divided respectively are called:

One degree (1°), one minute (1') and one second (1")

It means 1 right angle = 90° (90 degrees)

1 degree (1°) = 60' (60 minutes)

1 minute (1') = 60" (60 seconds)

In Trigonometry, mostly this system is used.

(ii) Centesimal System

This is called French System. In this system, the right angle is divided into 100 equal parts which are called grades. Each grade is divided into 100 minutes and each minute is divided into 100 seconds.

Parts so divided are respectively called:

One grade (1 g), one minute (1'), one second (1").

It means 1 right angle = 100 grades (100g)

1 grade (1 g) = 100 minutes (100')

1 minute (1') = 100 seconds (100")

90° = 100g (because each is a right angle)

This system is easier than Sexagesimal System. But to use this system many other systems will have to be devised that is why this system is not used.

(iii) Circular System

In this system, the unit of measuring angles is radian. It is that angle which is formed at the centre and is formed of an arc of length equal to radius in a circle.

There is one constant ratio between the circumference and dia of a circle. This is represented by π .

$$\frac{\text{Circumference}}{\text{Diameter}} = \text{constant point} = \pi$$

Circumference $= \pi \times$ dia

$= 2\pi r$ (where r is radius of the circle)

$$\pi = \frac{22}{7}$$

Circumference makes an angle $(2\pi r) = 360°$

Radius of the circle makes an angle (r) = 1 Radian

$$\text{ie : } \frac{C}{r} = \frac{360^\circ}{1\text{Radian}}$$

$$\frac{2\pi r}{r} = \frac{360^\circ}{1\text{Radian}}$$

$$2\pi = \frac{360^\circ}{1\text{Radian}}$$

2π Radian = 360°

π Radian = 180°

$$1 \text{ Radian} = \frac{180^\circ}{\pi}$$

$$1^\circ = \frac{\pi}{180^\circ} \text{ Radian}$$

Examples

1 Convert 45°36'20" into degree and decimal of degree.

60 second = 1 minute

$$20 \text{ second} = \frac{20}{60} = 0.333'$$

60 minute = 1 degree

$$36.333 \text{ minute} = \frac{36.333}{60} = 0.606^\circ$$

$45^0 36'20" = 45.606°$

2 Convert 24.59° into degree, minute and second

1 degree = 60 minute

0.59 degree = 0.59 x 60 = 35.4'

1 minute = 60 second

0.4 minute = 60 sec x 0.4

= 24"

Therefore $24.59° = 24^0 35'24"$

3 Change $50^0 37'30"$ into degrees

By changing angle degrees into decimals

$$30" = \frac{30}{60} = 0.50'$$

37'30" = 37.5'

$$37.5' = \frac{37.5}{60} = 0.625^0$$

$50^0 37'30" = 50.625^0$

4 Convert 23^0 25' 32" into radians

We know $1^0 = 60' = 3600"$

Therefore $23^0 25' 32"$

$$= \left(23 + \frac{25}{60} + \frac{32}{3600}\right) degrees$$

$$= \frac{82800 + 1500 + 32}{3600}$$

$$= \frac{84332}{3600}$$

But $180^0 = \pi$ radians

Therefore 23.4255 degrees

$$= \frac{23.4255}{180}\ \pi \text{ radians}$$

$$= \frac{23.4255}{180} \times \frac{22}{7} \text{ radians}$$

= 0.4089 radians

5 Convert $87^0$19' 57" into Radian.

$$19'57" = 19' + \frac{57'}{60}$$

$$= 19' + 0.95'$$

$$= 19.95'$$

$$87°19.95' = 87° + \frac{19.95^\circ}{60}$$

$$= 87° + 0.332° = 87.33°$$

$$1° = \frac{\pi}{180} \text{ radian}$$

$$87.33° = \frac{\pi}{180} \times 87.33 \text{ radian}$$

$$= 1.524 \text{ radian}$$

6 Convert 67°11'43" into Radian

$$11'43" = 11' + \frac{43'}{60}$$

$$= 11' + 0.716'$$

$$= 11.72'$$

$$67°11.72' = 67° + \frac{11.72^\circ}{60}$$

$$= 67° + 0.195°$$

$$= 67.2°$$

$$1° = \frac{\pi}{180} \text{ radian}$$

$$67.2° = \frac{\pi}{180} \times 67.2 \text{ radian}$$

$$= 1.173 \text{ radian}$$

7 Convert $\frac{4}{7}\pi$ radian into degrees

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree}$$

$$\frac{4}{7}\pi \text{ radian} = \frac{180}{\pi} \times \frac{4}{7}\pi \text{ degree}$$

$$= 102.9 \text{ degree}$$

$$= 102^0\ 0.9 \times 60'$$

$$= 102^0 54'$$

8 Convert 0.8357 radian into degrees

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree}$$

$$0.8357 \text{ radian} = \frac{180}{\pi} \times 0.8357 \text{ degree}$$

$$= 47.88^0$$

$$= 47^0\ 0.88 \times 60'$$

$$= 47^0 52.80'$$

$$= 47^0 52'0.8 \times 60"$$

$$= 47^0 52'48"$$

9 Convert 2.752 radian into degrees

$$1 \text{ Radian} = \frac{180}{\pi} \text{ degree}$$

$$2.7520 \text{ radian} = \frac{180}{\pi} \times 2.752 \text{ degree}$$

$$= 157.7^0$$

$$= 157.7^0 \times 60'$$

$$= 157^0 42'$$

10 Convent $\frac{3}{5}\pi$ radian into degrees

$$1 \text{ Radian} = \frac{180}{\pi} \text{ degree}$$

$$\frac{3}{5}\pi \text{ radian} = \frac{180}{\pi} \times \frac{3}{5}\pi \text{ degree}$$

$$= 108^0$$

Assignment

Convert into Degree

1 12 Radian

Convert into Radians

2 78^o

3 $47^0 20'$

4 $52^0 36' 45"$

5 $25^0 38"$

Convert into degree, minute and seconds

6 46.723^o

7 68.625^o

8 0.1269 Radians

9 2.625 Radians

10 3/5 Radians

Trigonometry - Trigonometrical ratios

Dependency

The sides of a triangle bear constant ratios for a given definite value of the angle. That is, increase or decrease in the length of the sides will not affect the ratio between them unless the angle is changed. These ratios are trigonometrical ratios. For the given values of the angle a value of the ratios

$\frac{BC}{AB}, \frac{AC}{AB}, \frac{BC}{AC}, \frac{AB}{BC}, \frac{AB}{AC}$ and $\frac{AC}{BC}$ do not change even when the sides AB, BC, AC are increased to AB', BC' and AC' or decreased to AB", BC" and AC".

For the angle

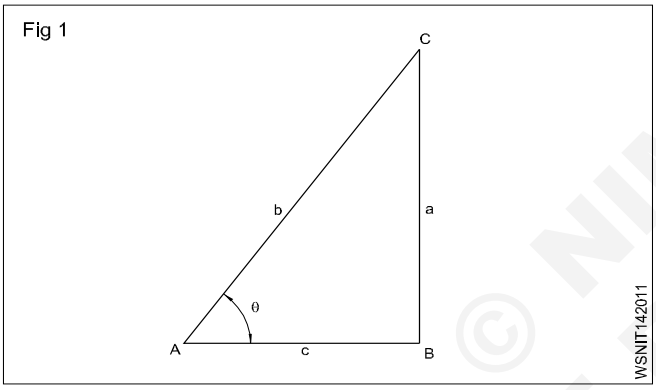
AC is the hypotenuse

AB is the adjacent side

BC is the opposite side.

The ratios

Fig 1



The six ratios between the sides have precise definitions.

$$\text{Sine } \theta = \frac{BC}{AC} = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \text{Sin } \theta$$

$$\text{Cosine } \theta = \frac{AB}{AC} = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \text{Cos } \theta$$

$$\text{Tangent } \theta = \frac{BC}{AB} = \frac{\text{Opposite side}}{\text{Adjacent side}} = \text{Tan } \theta$$

$$\text{Cosecant } \theta = \frac{AC}{BC} = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \text{Cosec } \theta$$

$$\text{Secant } \theta = \frac{AC}{AB} = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \text{Sec } \theta$$

$$\text{Cotangent } \theta = \frac{AB}{BC} = \frac{\text{Adjacent side}}{\text{Opposite side}} = \text{Cot } \theta$$

Relationship between the ratios

$$\text{Cosec } \theta = \frac{AC}{BC} = \frac{1}{\frac{BC}{AC}} = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{AC}{AB} = \frac{1}{\frac{AB}{AC}} = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{AB}{BC} = \frac{1}{\frac{BC}{AB}} = \frac{1}{\tan \theta}$$

$$\sin \theta = \frac{\text{side BC}}{\text{side AC}} = \frac{a}{b}$$

$$\cos \theta = \frac{\text{side AB}}{\text{side AC}} = \frac{c}{b}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{\frac{a}{b}}{\frac{c}{b}} = \frac{a}{b} \times \frac{b}{c} = \frac{a}{c}$$

$$= \frac{\text{side BC}}{\text{side AB}} = \tan \theta$$

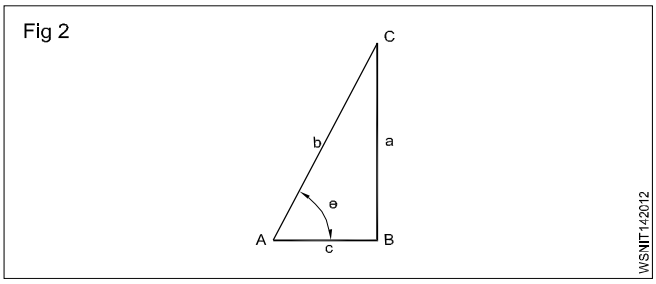
$$\sin \theta = \frac{1}{\text{cosec } \theta} \text{ or cosec } \theta = \frac{1}{\sin \theta} \text{ or } \sin \theta . \text{cosec } \theta = 1$$

$$\cos \theta = \frac{1}{\sec \theta} \text{ or } \sec \theta = \frac{1}{\cos \theta} \text{ or } \cos \theta . \sec \theta = 1$$

$$\tan \theta = \frac{1}{\cot \theta} \text{ or } \cot \theta = \frac{1}{\tan \theta} \text{ or } \cot \theta . \tan \theta = 1$$

By pythogoras theorem we have, $AC^2 = AB^2 + BC^2$

Fig 2



Dividing both sides of the equation by AC^2, we have

$$\frac{AC^2}{AC^2} = \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$$

$$= \left[\frac{AB}{AC}\right]^2 + \left[\frac{BC}{AC}\right]^2$$

$1 = (\cos \theta)^2 + (\sin \theta)^2$

$\sin^2 \theta + \cos^2 \theta = 1$

Sine, Cosine, Tangent, Cosec, Sec and Cotangent are the six trigonometrical ratios

$\tan \theta = \frac{\text{Sin } \theta}{\text{Cos } \theta}$ and $\sin^2 \theta + \cos^2 \theta = 1$

$\sin^2\theta + \cos^2\theta = 1$

It can be transformed as

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$\text{or } \cos^2\theta = 1 - \sin^2\theta$$

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$$

We know $\sin^2\theta + \cos^2\theta = 1$

Dividing both sides by $\cos^2\theta$.

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\text{or } 1 + \tan^2\theta = \sec^2\theta$$

Using the same equation

$$\sin^2\theta + \cos^2\theta = 1.$$

Dividing both sides by $\sin^2\theta$,

$$1 + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \text{cosec}^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

Trigonometrical Tables

Ratio	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

When θ increases,

Sine value increases;

Cosine value decreases;

Tangent value increases to more than 1 when the angle is more than 45° (tan60° = 1.732)

Sine of an angle = Cosine of its complementary angle

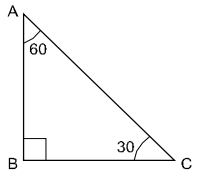
Cosine of an angle = Sine of its complementary angle

Examples

If $\sin 30^0 = \frac{1}{2}$ find the value of $\sin 60^0$

By applying pythagores theorem

Fig 3



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$BC^2 = AC^2 - AB^2$

$BC^2 = 2^2 - 1^2$

$= 4 - 1$

$= 3$

$BC = \sqrt{3}$

$\sin 60^0 = \frac{\sqrt{3}}{2}$

$\text{Cos}\theta = \frac{3}{5}$ Find the other trigonometrical ratios

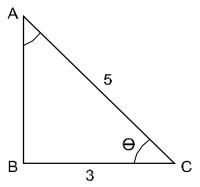
By applying pythagores theorem

$AB^2 = AC^2 - BC^2$

$= 5^2 - 3^2 \quad = 25 - 9$

$= 16$

Fig 4



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$AB = \sqrt{16} = 4$

Now $\sin\theta = \frac{4}{5}$

$\tan\theta = \frac{4}{3}$

$\text{Cosec } \theta = \frac{5}{4}$

$\sec\theta = \frac{5}{3}$

$\cot\theta = \frac{3}{4}$

Signs of trigonometrical functions for angles more than 90^0

Ratio	90 - θ	90 + θ	180 - θ	180 + θ	270 - θ	270 + θ	360 - θ	- θ
sin	cos	cos	sin	- sin	- cos	- cos	- sin	- sin
cos	sin	- sin	- cos	- cos	- sin	sin	cos	cos
tan	cot	- cot	- tan	tan	cot	- cot	- tan	- tan
cosec	sec	sec	cosec	- cosec	- sec	- sec	- cosec	- cosec
sec	cosec	- cosec	- sec	- sec	- cosec	cosec	sec	sec
cot	tan	- tan	- cot	cot	tan	- tan	- cot	- cot

Simplify :

cot θ + tan (180+θ) + tan(90-θ) + (tan 360 - θ)

= cot θ + tan θ - cot θ - tan θ

= 0

Simplify :

$$\frac{\cos(90+\theta)\sec(-\theta)\tan(180-\theta)}{\sec(360-\theta)\sin(180+\theta)\cos(90-\theta)}$$

$$= \frac{(-\sin\theta)\,x\,(\sec\theta)\,x\,(-\tan\theta)}{(\sec\theta)\,x\,(-\sin\theta)\,x\,(-\sin\theta)}$$

$$= \frac{\tan\theta}{\sin\theta} = \frac{1}{\cos\theta} = \sec\theta$$

simplify:

$$\frac{\cos(90^\circ+\theta)\sec(-\theta)\tan(180^\circ-\theta)}{\sec(360^\circ-\theta)\sin(180^\circ+\theta)\cot(90^\circ-\theta)}$$

cos (90^0 + θ) = - sin θ

sec (- θ) = sec θ

tan (180^0 - θ) = - tan θ

sec (360^0 - θ) = sec θ

sin (180^0 + θ) = - sin θ

cot (90^0 + θ) = - tan θ

$$\frac{\cos(90^\circ+\theta)\sec(-\theta)\tan(180^\circ-\theta)}{\sec(360^\circ-\theta)\sin(180^\circ+\theta)\cot(90^\circ-\theta)}$$

$$= \frac{(-\sin\theta)(\sec\theta)(\tan\theta)}{(\sec\theta)(-\sin\theta)(-\tan\theta)}$$

= 1

Simplify:

Cot θ + tan (180^0 + θ) + tan (90^0 + θ) + tan (360^0 - θ)

tan (180^0 - θ) = tan θ

tan (90^0 + θ) = - cot θ

tan (360^0 - θ) = - tan θ

cot θ + tan (180^0 + θ) + tan (90^0 + θ) + tan (360^0 - θ)

cot θ + tan θ - cot θ - tan θ = 0

Assignment

1 Given sin 30^0 = 1/2, find the value of tan 60^0

2 If cos θ = 4/5, find the other radios

3 If sin A = 3/5, find cos θ, tan θ & sec θ

4 If tan θ = 24/7, find sin θ and cos θ

5 Find the value of cos θ and tan θ, if sin θ = 1/2

6 If cos θ = 5/13, find the value of tan θ

7 If sin θ = 1/2, find the value of $\sin^2\theta - \cos^2\theta$

8 What is the value of

$$\frac{\sin^2 30^\circ}{\cos^2 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ}$$

Simplify :

1 tan (90 + A) + (tan 180 + A) tan (90 + A)

2 $$\frac{\cos(90+\theta)\cdot\sec(-\theta)\cdot\tan(180-\theta)}{\sec(360+\theta)\cdot\sin(180+\theta)\cdot\cot(90+\theta)}$$

Trigonometry - Trigonometrical tables

Use of trigonometrical tables (Ref: Sin, Cos & Tan Table)

	Minutes from 0 to 4						Mean difference				
Deg.	0'	6'	12'	18'	24'	54'	1'	2'	3'	4'	5'
0				.				.			
1				.				.			
2				.				.			
3				.				.			
.				.				.			
.				.				.			
.				.				.			
26	..	..	..	x	..	..	..	5			
.											
.											
.											
89											

Sine value for 26°– 20'

Refer to Natural sine table.

Degrees column go up to 26° down

Minutes column 18' horizontal and under this note the value which is given as 0.4431.

Under mean difference for 2' in the same horizontal line 5 is given. Add this to the extreme right number noted for 26°– 18'.

Sine 26° – 20' = 0.4431 + .0005 = 0.4436

Cosine value for 43° – 41'

Referring to the Natural cosines table for 43° –36' it is given as 0.7242 and the mean difference for 5' minutes is given as 10.

cos 43°– 41' = value for cos 43°.36 '

– the value given for

mean difference of 5 ' = 0.7242 – 0.0010

= 0.7232

When reading sine value add the mean difference value. When reading cosine value subtract the mean difference value.

Arrangement

Values of trigonometrical ratios can be taken from mathematical tables.

The left hand vertical column consists of degrees.

The top horizontal column is arranged in minutes in steps of 6' from 0' to 54'. In the extreme right horizontal columns the mean differences are written in minutes from 1' to 5' in steps of 1' to account for angles with minutes between the interval of 6'.

- **The values of cosine, cosecant and cotangent decrease when the value of the angle increases.**
- **For sine, secant and tangent, the value increases when the angle increases.**
- **The value of sine and cosine will never be more than 1.**
- **The value of secant and cosecant will never be less than 1.**
- **The value of Tan and Cot ranges from 0 to ∞.**

EXAMPLE

From the tables obtain the cosine of 45°–20 '.

cos 45°–18 ' = 0.7108

mean difference for 2 ' = 0.0004

cos 45°– 20 ' = 0.7104

— — — — — — —

SINE TABLE

1 Sin 25^0 = 0.4226

23 Sin 17^0 5'

sin 17°	=	0.2924
Difference 5'	=	14
sin 17° 5'	=	0.2938 Ans

3 **sin 17° 45' 13"**

sin 17° 46'	=	0.3051
sin17° 45'	=	0.3048
Difference 1'	=	0.0003

$1'(or)\,60'' = 0.0003$

$13'' = \frac{0.0003}{60} \times 13$

$= \frac{0.0039}{60}$

$= \frac{0.00039}{6}$

$= 0.000065$

$\sin 17^0\,45' = 0.3048$

$13'' = 0.000065$

$\sin 17^0\,45'13'' = 0.304865$ Ans.

4 $\sin 82^\circ\ 14'18''$

$\sin 82^\circ\ 15' = 0.9908$

$\sin 82^\circ\ 14' = 0.9908$

Difference $1' = 0$

$1'(or)\,60'' = 0$

$\sin 18'' = 0$

$\sin 82^\circ\ 14' = 0.9908$

$18'' = 0.0000$

$\sin 82^\circ 41'18'' = 0.9908$ Ans.

Finding the corresponding angles when sine values are given:

1. Sin θ = 0.9925

θ = 83^0

2. Sin θ = 0.8791

$0.8788 = \text{Sin } 61^0\ 30'$

$0.0003 = 2'$

$0.8791 = \text{Sin } 61^0\ 32'$

3. sin θ = 0.68015

$0.6794 = \sin 42^0\,48'$

$0.0006 = 3'$

$0.6800 = \sin 42^0\,51'$

$0.6803 = \sin 42^0\,52'$

Difference 0.0003 = 1' (or) 60"

$0.00015 = \frac{60}{0.0003} \times 0.00015$

$= \frac{60 \times 15}{30}$

$= 30''$

$0.6800 = \sin 42^0\,51'$

$0.00015 = 30''$

$0.68015 = \sin 42^0\,51'30''$

θ = 42^0 51'30"

Calculations involving tapers

D - Big diameter of the taper

d - small diameter of the taper

C - Taper Ratio - 1:x

Fig 1

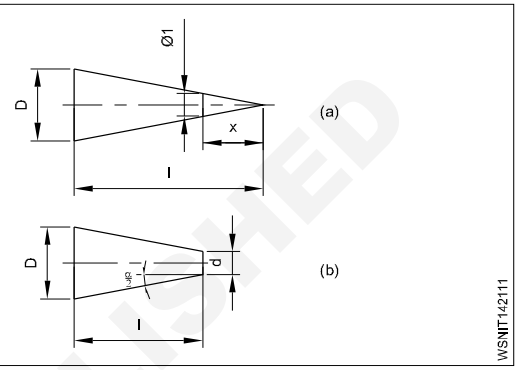

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Fig 2

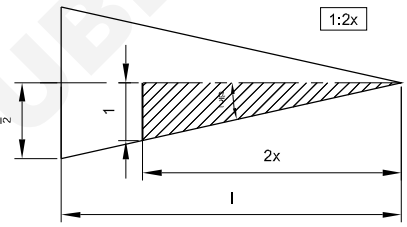

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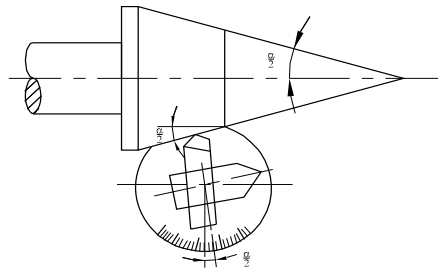
Fig 3



WSNIT142113

$\frac{C}{2}$ Ratio of inclination - 1:2 **x**

l - length of taper

a - included angle of taper

$\frac{\alpha}{2}$ - setting angle

Taper ratio = Ratio of inclination (for wedges).

Taper ratio

The ratio between the difference in diameter to the length of the taper is known as taper ratio. *D* is the difference in larger diameter shown in the sketch as the small diameter of taper is 0. Taper ratio is *D : l* . In the sectioned portion the difference in diameter is 1 and the length of taper is shown as *x* .

C = D: d = 1 : x as per Fig 1 (a), C = $\frac{D-d}{l}$ as per Fig 1 (b)

Ratio of inclination

Taking half of the taper, $\frac{D}{2}$ is the difference in diameter for a taper length of l, if d = 0.

$\therefore \frac{C}{2} = \frac{D}{2l}$ if the small diameter is 0

or $\frac{C}{2} = \frac{D-d}{2l}$

1 Ratio of inclination = $\frac{1}{2}$ of the taper ratio.

Setting angle

One of the methods of turning taper is by swivelling the compound slide to an angle known as setting angle and feeding the tool at an angle to the axis of work.

$\tan\frac{\alpha}{2} = \frac{C}{2} = \frac{D-d}{2l}$

$\tan\frac{\alpha}{2} = \frac{\text{taper ratio}}{2}$

$= \frac{\text{difference in diameter}}{2 \times \text{taper length}}$

NOTE: $\frac{\alpha}{2}$ is the setting angle which is equal to half of the included angle of the taper.

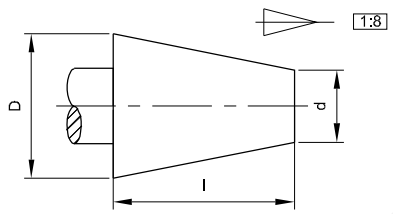
Taper Ratio $C = 1 : x$ or $D : l$ or $(D-d) : l$

Ratio of inclination $\frac{C}{2} = 1 : 2x = \frac{D}{2} : l$ or $\left(\frac{D-d}{2}\right)$:l

Setting angle determination is by the formula

$\tan\frac{\alpha}{2} = \frac{D-d}{2l} = \frac{C}{2}$

Fig 4



EXAMPLE

A pivot in the form of a frustum of a cone has a taper ratio 1:8. If the small diameter is 30 mm and length of taper is 80 mm, find its large diameter.

C = 1:8 = $\frac{1}{8}$

$\therefore \frac{D-d}{l} = \frac{1}{8}$

$\therefore D - d = \frac{1}{8} = \frac{80}{8} = 10\,\text{mm}.$

D – 30 mm = 10 mm

D = 10 mm + 30 mm = 40 mm

Large diameter D = 40 mm

— — — — — — — —

Cos Table

1 Cos 38°

Cos 38° = 0.7880

2 Cos 83°12'

Cos 83°12' = 0.1184

3 Cos 26°40'

Cos 26°36' = 0.8942

4' = 5 (-)

Cos 26°40' = 0.8937

---

4 Cos 31°20'

Cos 31°18' = 0.8545

2' = 3 (-)

Cos 31°20' = 0.8542

Find the corresponding angles when cos values are given:

1 Cos θ = 0.5150

θ = 59°

2 Cos θ = 0.0192

θ = 88°54'

3 Cos θ = 0.9682

0.9686 = cos 14°24'

(-) 4 = 5'

0.9682 = cos 14°29'

θ = 14°29'

4 Cos θ = 0.8476

0.8480 = cos 32° 0'

(-) 0.0003 = 2'

0.8477 = cos 32° 2'

0.8475 = cos 32° 3'

0.0002 = 1' (or) 60"

$$0.0001 = \frac{60}{0.0002} \times 0.0001$$

$$= \frac{60}{2} \times 1$$

= 30"

0.8477 = Cos 32° 2'

(-) 0.0001 = 30" (+)

0.8476 = Cos 32° 2' 30"

---

tan Table

1 tan 35° 37'

tan 35° 36' = 0.7159

1' = 0.0004

---

tan 35° 37' = 0.7163

---

2 tan 50° 5'

Tan 50° 0' = 1.1918

5' = 0.0036

Tan 50° 5' = 1.1954

Find the corresponding angles when tan values are given

1 tan θ = 0.3972

0.3959 = tan 21° 36'

0.0013 = 4'

0.3972 = tan 21° 40'

---

2 tan θ = 1.0065

1.0035 = tan 45° 6'

0.0030 = 5'

1.0065 = tan 45° 11'

Problems Related with Trigonometrical tables

1 A 250 mm Sine bar is used to measure an angle. If the difference in height is 5 cm, find the angle.

$$Sin\theta = \frac{\text{Opp.side}}{\text{Hyp.}} = \frac{h}{l}$$

$$= \frac{5 \text{ cm}}{250 \text{ mm}}$$

$$= \frac{50 \text{ mm}}{250 \text{ mm}}$$

= 0.2000

θ = 11° 32'

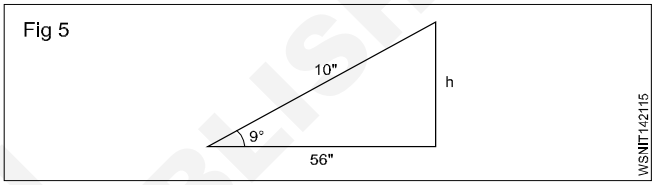
2 Find the height of the slip gauge if a Sine bar with plugs of 10" centre is set up to inspect a taper having an included angle of 9° 56".

$$Sin\theta = \frac{\text{Opp.side}}{\text{Hyp.}} = \frac{h}{l}$$

$$\text{Sin } 9°56" = \frac{h}{10}$$

$\therefore h = 10 \times \sin 9°56"$

Fig 5



Sin 9° 0" = 0.1564

Sin 9° 1" = 0.1567

1' (or) 60" = 0.0003

$$56" = \frac{0.0003}{60} \times 56$$

$$= \frac{0.0168}{60}$$

$$= \frac{0.00168}{6}$$

= 0.00028

sin9° 0' 00" = 0.1564

56" = 0.00028

---

sin9° 0' 56" = 0.15668

---

h = 10 x sin9° 56"

= 10 x 0.15668

= 1.5668 cm

Height of slip gauge = 1.5668"

3 Find the angle which the ladder makes with the ground if the foot of a 4.5 m long ladder is placed at 1 m away from the wall.

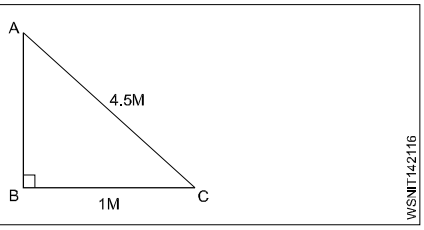
In Right angled D

$$\text{Cos C} = \frac{BC}{AC}$$

$$\text{Cos}\ \theta = \frac{1\ \text{m}}{4.5\ \text{m}} = 0.2222$$

	Cos	=	0.2222
	0.2233	=	cos 77° 6'
(-)	0.0011	=	4' (+)
	0.2222	=	cos 77° 10'
	θ	**=**	**77° 10'**

Fig 6



Assignment

I Find the values of the given angles

1 Sin 65°
2 Sin 42° 23'
3 Sin 66° 35' 32"
4 Sin 7° 15' 41"
5 Sin 27° 27"
6 Cos 47° 39'
7 Cos 47° 39'
8 Cos 79° 31' 53"
9 Tan 28° 45'
10 Tan 67° 27' 36"

II Find corresponding angles for given values

1 Sin θ = 0.3062
2 Sin θ = 0.6002
3 Sin θ = 0.22453
4 Sin θ = 0.04802
5 Cos θ = 0.6446
6 Cos θ = 0.8926
7 Cos θ = 0.11773
8 Cos θ = 0.21646
9 Tan θ = 0.3411
10 Tan θ = 2.3868

III

1 Calculate its base. if the slant height of a cone is 12.25 cm and the vertex angle is 110^0.
2 A ladder 2.5 m long makes an angle of 60^0 with the ground. Find the height of the wall where the ladder touches the wall.
3 A sine bar of 200 mm is to be set at an angle of $15^0 15'3"$. Select the slip gauge block to built up the required height.
4 In a right angled triangle ABC, ∠C = 90°, If AB = 50 mm and ∠B = 75°, Find the remaining sides.
5 Calculate the required length of the bar for this point if a centre point having an included angle of 60° is to be turned at the end of a 50 mm dia bar.

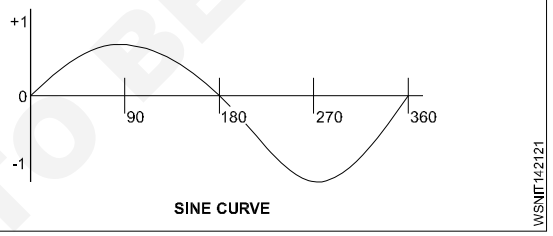
Natural Sines

°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
0	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	3	6	9	12	15
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	3	6	9	12	15
2	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506	3	6	9	12	15
3	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680	3	6	9	12	15
4	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854	3	6	9	12	14
5	0.0872	0.0899	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028	3	6	9	12	14
6	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201	3	6	9	12	14
7	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374	3	6	9	12	14
8	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547	3	6	9	11	14
9	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719	3	6	9	11	14
10	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891	3	6	9	11	14
11	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062	3	6	9	12	14
12	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232	3	6	9	11	14
13	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402	3	6	8	11	14
14	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571	3	6	8	11	14
15	0.2558	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740	3	6	8	11	14
16	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907	3	6	8	11	14
17	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074	3	6	8	11	14
18	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239	3	6	8	11	14
19	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404	3	5	8	11	14
20	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567	3	5	8	11	14
21	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730	3	5	8	11	14
22	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891	3	5	8	11	13
23	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051	3	5	8	11	13
24	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210	3	5	8	11	13
25	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368	3	5	8	11	13
26	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524	3	5	8	10	13
27	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679	3	5	8	10	13
28	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833	3	5	8	10	13
29	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985	3	5	8	10	13
30	0.500	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135	3	5	8	10	13
31	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284	2	5	7	10	12
32	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432	2	5	7	10	12
33	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577	2	5	7	10	12
34	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721	2	5	7	10	12
35	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864	2	5	7	9	12
36	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004	2	5	7	9	12
37	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143	2	5	7	9	12
38	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280	2	5	7	9	11
39	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414	2	4	7	9	11
40	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547	2	4	7	9	11
41	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678	2	4	7	9	11
42	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807	2	4	6	9	11
43	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0,6921	0.6934	2	4	6	8	11
44	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059	2	4	6	8	10
45	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181	2	4	6	8	10
46	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302	2	4	6	8	10
47	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420	2	4	6	8	10
48	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536	2	4	6	8	10
49	0.7547	0.7558	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649	2	4	6	8	9
50	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760	2	4	6	7	9
51	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869	2	4	5	7	9
52	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976	2	4	5	7	9
53	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080	2	3	5	7	9
54	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181	2	3	5	7	8
55	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281	2	3	5	7	8
56	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377	2	3	5	6	8
57	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471	2	3	5	6	8
58	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563	2	3	5	6	8
59	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652	1	3	4	6	7

Natural Sines

°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
60	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738	1	3	4	6	7
61	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821	1	3	4	6	7
62	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902	1	3	4	5	7
63	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980	1	3	4	5	6
64	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056	1	3	4	5	6
65	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128	1	2	4	5	6
66	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198	1	2	3	5	6
67	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265	1	2	3	4	6
68	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330	1	2	3	4	5
69	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391	1	2	3	4	5
70	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449	1	2	3	4	5
71	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505	1	2	3	4	5
72	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558	1	2	3	3	4
73	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608	1	2	2	3	4
74	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655	1	2	2	3	4
75	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699	1	1	2	3	4
76	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740	1	1	2	3	3
77	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778	1	1	2	2	3
78	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813	1	1	2	2	3
79	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845	1	1	2	2	3
80	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874	0	1	1	2	2
81	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900	0	1	1	2	2
82	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923	0	1	1	1	2
83	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943	0	1	1	1	2
84	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960	0	1	1	1	1
85	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974	0	0	1	1	1
86	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985	0	0	1	1	1
87	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993	0	0	0	1	1
88	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998	0	0	0	0	0
89	0.9998	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0
90	1.0000														

Fig 1



SINE CURVE

Quadrant	Angle	sinA =	Examples
First	0 to 90°	sin A	sin 34°38' = 0.5683
Second	90° to 180°	sin(180° – A)	sin 145°22' = sin(180° – 145° 22') = sin 34°38' = 0.5683
Third	180° to 270°	–sin(A – 180°)	sin 214°38' = –sin(214°38' – 180°) = –sin34°38' = –0.5683
Fourth	270° to 360°	–sin(360° – A)	sin 325°22' = –sin(360° – 325°22') = – sin 34°38' = –0.5683

Natural Cosines

Numbers in different columns to be subtracted**, not added**

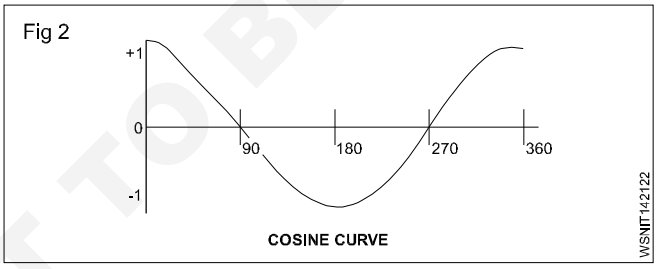
°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0	0	0	0	0
1	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995	0	0	0	0	0
2	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987	0	0	0	1	1
3	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977	0	0	1	1	1
4	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963	0	0	1	1	1
5	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947	0	1	1	1	1
6	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928	0	1	1	1	2
7	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905	0	1	1	1	2
8	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880	0	1	1	2	2
9	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851	0	1	1	2	2
10	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820	1	1	2	2	3
11	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785	1	1	2	2	3
12	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748	1	1	2	2	3
13	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707	1	1	2	3	3
14	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664	1	1	2	3	4
15	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617	1	2	2	3	4
16	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568	1	2	2	3	4
17	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516	1	2	3	3	4
18	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461	1	2	3	4	5
19	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403	1	2	3	4	5
20	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342	1	2	3	4	5
21	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278	1	2	3	4	5
22	0.9272	0.9625	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212	1	2	3	4	6
23	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143	1	2	3	5	6
24	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070	1	2	4	5	6
25	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996	1	3	4	5	6
26	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918	1	3	4	5	6
27	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838	1	3	4	5	7
28	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755	1	3	4	6	7
29	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0.8678	0.8669	1	3	4	6	7
30	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581	1	3	4	6	7
31	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490	2	3	5	6	8
32	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396	2	3	5	6	8
33	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300	2	3	5	6	8
34	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202	2	3	5	7	8
35	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100	2	3	5	7	8
36	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997	2	3	5	7	9
37	0.7986	0.7976	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891	2	4	5	7	9
38	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782	2	4	5	7	9
39	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672	2	4	6	7	9
40	0.7660	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559	2	4	6	8	9
41	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0.7455	0.7443	2	4	6	8	10
42	0.7431	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325	2	4	6	8	10
43	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206	2	4	6	8	10
44	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083	2	4	6	8	10
45	0.7071	0.7059	0.7046	0,7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959	2	4	6	8	10
46	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833	2	4	6	8	11
47	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704	2	4	6	9	11
48	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574	2	4	7	9	11
49	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441	2	4	7	9	11
50	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307	2	4	7	9	11
51	0.6293	0.6280	0.6266	0.6252	0.6239	0.6255	0.6211	0.6198	0.6184	0.6170	2	5	7	9	11
52	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6404	0.6060	0.6046	0.6032	2	5	7	9	12
53	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892	2	5	7	9	12
54	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750	2	5	7	9	12
55	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606	2	5	7	10	12
56	0.5592	0.5577	0.5563	0.5548	0.5534	0.5519	0.5505	0.5490	0.5476	0.5461	2	5	7	10	12
57	0.5446	0.5432	0.5417	0.5402	0.5388	0.5373	0.5358	0.5344	0.5329	0.5314	2	5	7	10	12
58	0.5299	0.5284	0.5270	0.5255	0.5240	0.5225	0.5210	0.5195	0.5180	0.5165	2	5	7	10	12
59	0.5150	0.5135	0.5120	0.5105	0.5090	0.5075	0.5060	0.5045	0.5030	0.5015	3	5	8	10	13

Natural Cosines

Numbers in different columns to be subtracted**, not added**

°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
60	0.5000	0.4985	0.4970	0.4955	0.4939	0.4924	0.4909	0.4894	0.4879	0.4863	3	5	8	10	13
61	0.4848	0.4833	0.4818	0.4802	0.4787	0.4772	0.4756	0.4741	0.4726	0.4710	3	5	8	10	13
62	0.4695	0.4679	0.4664	0.4648	0.4633	0.4617	0.4602	0.4586	0.4571	0.4555	3	5	8	10	13
63	0.4540	0.4524	0.4509	0.4493	0.4478	0.4462	0.4446	0.4431	0.4415	0.4399	3	5	8	10	13
64	0.4384	0.4368	0.4352	0.4337	0.4321	0.4305	0.4289	0.4274	0.4258	0.4242	3	5	8	11	13
65	0.4226	0.4210	0.4195	0.4179	0.4163	0.4147	0.4131	0.4115	0.4099	0.4083	3	5	8	11	13
66	0.4067	0.4051	0.4035	0.4019	0.4003	0.3987	0.3971	0.3955	0.3939	0.3923	3	5	8	11	13
67	0.3907	0.3891	0.3875	0.3859	0.3843	0.3827	0.3811	0.3795	0.3778	0.3762	3	5	8	11	13
68	0.3746	0.3730	0.3714	0.3697	0.3681	0.3665	0.3649	0.3633	0.3616	0.3600	3	5	8	11	14
69	0.3584	0.3567	0.3551	0.3535	0.3518	0.3502	0.3486	0.3469	0.3453	0.3437	3	5	8	11	14
70	0.3420	0.3404	0.3387	0.3371	0.3355	0.3338	0.3322	0.3305	0.3289	0.3272	3	5	8	11	14
71	0.3256	0.3239	0.3223	0.3206	0.3190	0.3173	0.3156	0.3140	0.3123	0.3107	3	6	8	11	14
72	0.3090	0.3074	0.3057	0.3040	0.3024	0.3007	0.2990	0.2974	0.2957	0.2940	3	6	8	11	14
73	0.2924	0.2907	0.2890	0.2874	0.2857	0.2840	0.2823	0.2807	0.2790	0.2773	3	6	8	11	14
74	0.2756	0.2740	0.2723	0.2706	0.2689	0.2672	0.2656	0.2639	0.2622	0.2605	3	6	8	11	14
75	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0.2436	3	6	8	11	14
76	0.2419	0.2402	0.2385	0.2368	0.2351	0.2334	0.2317	0.2300	0.2284	0.2267	3	6	8	11	14
77	0.2250	0.2233	0.2215	0.2198	0.2181	0.2164	0.2147	0.2130	0.2113	0.2096	3	6	9	11	14
78	0.2079	0.2062	0.2045	0.2028	0.2011	0.1994	0.1977	0.1959	0.1942	0.1925	3	6	9	11	14
79	0.1908	0.1891	0.1874	0.1857	0.1840	0.1822	0.1805	0.1788	0.1771	0.1754	3	6	9	11	14
80	0.1736	0.1719	0.1702	0.1685	0.1668	0.1650	0.1633	0.1616	0.1599	0.1582	3	6	9	11	14
81	0.1564	0.1547	0.1530	0.1513	0.1495	0.1478	0.1461	0.1444	0.1426	0.1409	3	6	9	11	14
82	0.1392	0.1374	0.1357	0.1340	0.1323	0.1305	0.1288	0.1271	0.1253	0.1236	3	6	9	12	14
83	0.1219	0.1201	0.1184	0.1167	0.1149	0.1132	0.1115	0.1097	0.1080	0.1063	3	6	9	12	14
84	0.1045	0.1028	0.1011	0.0993	0.0976	0.0958	0.0941	0.0924	0.0906	0.0889	3	6	9	12	14
85	0.0872	0.0854	0.0837	0.0819	0.0802	0.0785	0.0767	0.0750	0.0732	0.0715	3	6	9	12	14
86	0.0698	0.0680	0.0663	0.0645	0.0628	0.0610	0.0593	0.0576	0.0558	0.0541	3	6	9	12	15
87	0.0523	0.0506	0.0488	0.0471	0.0454	0.0436	0.0419	0.0401	0.0384	0.0366	3	6	9	12	15
88	0.0349	0.0332	0.0314	0.0297	0.0279	0.0262	0.0244	0.0227	0.0209	0.0192	3	6	9	12	15
89	0.0175	0.0157	0.0140	0.0122	0.0105	0.0087	0.0070	0.0052	0.0035	0.0017	3	6	9	12	15
90	0.0000														

Fig 2



COSINE CURVE

Quadrant	Angle	cos A =	Examples
First	0 to 90°	cos A	cos 33°26' = 0.8345
Second	90° to 180°	–cos(180° – A)	sin 146°34' = –cos(180° – 146o 34') = –cos 33o26' = –0.8345
Third	180° to 270°	cos(A – 180°)	cos 213°26' = –cos(213°26' – 180°) = –cos 33°26' = –0.8345
Fourth	270° to 360°	cos(360° – A)	cos 326°34' = cos(360° – 326°34') = cos 33°26' = 0.8345

Natural Tangents

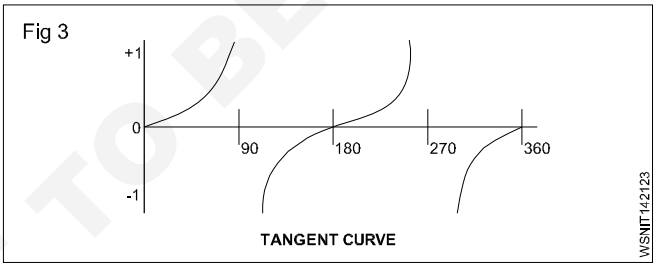
°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
0	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	3	6	9	12	15
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	3	6	9	12	15
2	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507	3	6	9	12	15
3	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682	3	6	9	12	15
4	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857	3	6	9	12	15
5	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033	3	6	9	12	15
6	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210	3	6	9	12	15
7	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388	3	6	9	12	15
8	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566	3	6	9	12	15
9	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745	3	6	9	12	15
10	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926	3	6	9	12	15
11	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107	3	6	9	12	15
12	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290	3	6	9	12	15
13	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475	3	6	9	12	15
14	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661	3	6	9	12	16
15	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849	3	6	9	13	16
16	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038	3	6	9	13	16
17	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230	3	6	10	13	16
18	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424	3	6	10	13	16
19	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620	3	7	10	13	16
20	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819	3	7	10	13	17
21	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020	3	7	10	13	17
22	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224	3	7	10	14	17
23	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431	3	7	10	14	17
24	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642	4	7	11	14	18
25	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856	4	7	11	14	18
26	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073	4	7	11	15	18
27	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295	4	7	11	15	18
28	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520	4	8	11	15	19
29	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750	4	8	12	15	19
30	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985	4	8	12	16	20
31	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224	4	8	12	16	20
32	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469	4	8	12	16	20
33	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720	4	8	13	17	21
34	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976	4	9	13	17	21
35	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239	4	9	13	17	22
36	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508	5	9	14	18	23
37	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785	5	9	14	18	23
38	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069	5	9	14	19	24
39	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361	5	10	15	20	24
40	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662	5	10	15	20	25
41	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972	5	10	16	21	26
42	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293	5	11	16	21	27
43	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623	6	11	17	22	28
44	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965	6	11	17	23	28
45	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319	6	12	18	24	30
46	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686	6	12	18	25	31
47	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067	6	13	19	25	32
48	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463	7	13	20	27	33
49	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875	7	14	21	28	34
50	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305	7	14	22	29	36
51	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753	8	15	23	30	38
52	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222	8	16	24	31	39
53	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713	8	16	25	33	41
54	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229	9	17	26	34	43
55	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770	9	18	27	36	45
56	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340	10	19	29	38	48
57	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941	10	20	30	40	50
58	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577	11	21	32	43	53
59	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251	11	23	34	45	56

Natural Tangents

°	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	1'	2'	3'	4'	5'
60	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966	12	24	36	48	60
61	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728	13	26	38	51	64
62	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542	14	27	41	55	68
63	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413	15	29	44	58	73
64	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348	16	31	47	63	78
65	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355	17	34	51	68	85
66	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445	18	37	55	73	92
67	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627	20	40	60	79	99
68	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916	22	43	65	87	108
69	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326	24	47	71	95	119
70	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878	26	52	78	104	131
71	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595	29	58	87	116	145
72	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506	32	64	96	129	161
73	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646	36	72	108	144	180
74	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062	41	81	122	163	204
75	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812	46	93	139	186	232
76	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972	53	107	160	213	267
77	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646					
78	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970					
79	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140					
80	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432					
81	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264					
82	7.1154	7.2066	7.3002	6.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285					
83	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572					
84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20					
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95					
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46					
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27					
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08					
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0					
90	×														

DIFFERENCES UNTRUSTWORTHY HERE

Fig 3



TANGENT CURVE

Quadrant	Angle	tan A =	Examples
First	0 to 90°	tan A	tan 56°17' = 1.4986
Second	90° to 180°	–tan(180° – A)	tan 123°43' = –tan(180° – 123° 43') = –tan 56°17' = –1.4986
Third	180° to 270°	tan(A – 180°)	tan 236°17' = tan(236°17' – 180°) = tan 56°17' = 1.4986
Fourth	270° to 360°	–tan(360° – A)	tan 303°43' = –tan(360° – 303°43') = – tan 56°17' = –1.4986